Training Program for NET in Physics (TPNP)

ASSIGNMENT 2

1. A particle of mass m moves in two dimensions in an anisotropic harmonic oscillator potential $V(x,y) = \frac{1}{2}m\omega^2x^2 + 2m\omega^2y^2$. The energy eigenvalues are given by
(a) $\hbar\omega(2n+1)$, (b) $\hbar\omega(n+1)$, (c) $2\hbar\omega(n+1)$, (d) $\hbar\omega(n+\frac{3}{2})$,
where n is a positive integer or zero.

2. Consider a three dimensional isotropic harmonic oscillator with Hamiltonian

$$H = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + \frac{1}{2}m\omega^2(x^2 + y^2 + z^2).$$

The number of distinct eigenstates with energy eigenvalue $\frac{5}{2}\hbar\omega$ is
(a) 5, (b) 3, (c) 2, (d) 1.

3. The lowest energy level of an electron placed in an infinite square well potential is 2.35 eV. The next two energy levels are
(a) 4.7 eV, 7.05 eV; (b) 9.4 eV, 14.1 eV; (c) 9.4 eV, 21.15 eV;
(d) 7.05 eV, 21.15 eV.

4. The lowest energy level of an electron placed in an infinite square well potential of width a is 4 eV. If the width is reduced to $a/2$, the lowest energy level will become
(a) 1 eV (b) 2 eV, (c) 8 eV, (d) 16 eV.

5. Consider a one-dimentional infinite square well potential

$$V(x) = \begin{cases} 0, & 0 \le x \le L \\ \infty, & \text{otherwise} \end{cases}$$

If each of the lowest two energy levels are occupied by identical non-interacting Bosons (one in each level), then the unnormalized wave function of the combined system is
(a) $\psi(x_1,x_2) = \sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}$
(b) $\psi(x_1,x_2) = \sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L} - \sin\frac{2\pi x_1}{L}\sin\frac{\pi x_2}{L}$
(c) $\psi(x_1,x_2) = \sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L} + \sin\frac{2\pi x_1}{L}\sin\frac{\pi x_2}{L}$
(d) $\psi(x_1,x_2) = \sin\frac{\pi x_1}{L}\sin\frac{\pi x_2}{L} + \sin\frac{2\pi x_1}{L}\sin\frac{2\pi x_2}{L}$

6. The ground state energy of the hydrogen atom is -13.6 eV. The energy of the second excited state is
(a) -27.2 eV, (b) -6.8 eV, (c) -1.51 eV, (d) -4.5 eV.

7. The ground state energy of hydrogen atom is -13.6 eV. Calculate the photon energy corresponding to the first Balmer line in the hydrogen spectrum. Using the following data
1 eV = 1.602×10^{-19} Joules, Planck's Constant $h = 6.626 \times 10^{-34}$ Joules sec., velocity of light $c = 3 \times 10^{8}$ m/sec.,
the wavelength of the emitted photon obtained is
(a) 122 n.m. (b) 656 n.m. (c) 1875 n.m. (d) 4050 n.m.

8. Consider a system of two particles of masses m_1 and m_2 separated by a fixed distance r_0 rotating about an axis passing through its centre of mass (c.m.) and normal to the plane containing the two particles. If I is the moment of inertia of such a system, the energy level corresponding to the orbital angular momentum quantum number $l = 2$ is given by
(a) $\frac{\hbar^2}{2I}$, (b) $\frac{\hbar^2}{I}$, (c) $\frac{2\hbar^2}{I}$, (d) $\frac{3\hbar^2}{I}$.

9. The expectation value of x^2 for a particle in a one-dimensional infinite square well potential of width $2a$ which is symmetrical about $x = 0$ is
(a) $\frac{a^2}{3}$, (b) $\frac{a^2}{3} - \frac{2a^2}{n^2\pi^2}$, (c) $\frac{a^2}{3} - \frac{2a^2}{\pi^2}$, (d) $\frac{a^2}{3} - \frac{2a^2}{n^2}$.

10. Positronium is an atom formed by an electron and a positron. The mass of a positron is the same as that of an electron and its charge is equal in magnitude but opposite in sign to that of an electron. The positronium atom is thus similar to the hydrogen atom with the positron replacing the proton. If the binding energy of the hydrogen atom is 13.6 eV, the binding energy of the positronium atom is
(a) 13.6 eV, (b) 6.8 eV, (c) 27.2 eV (d) 3.4 eV.

11. The positronium atom exists in two spin states $S = 0$ and $S = 1$ and it can annihilate into
(a) one photon, (b) two or three photons, (c) two photons if $S = 0$ and three photons if $S = 1$, (d) two photons if $S = 1$ and three photons if $S = 0$.

12. For a particle with spin $S = 1$, the states $|m\rangle$, $m = -1, 0, 1$ are the eigenstates of the z component of the spin angular momentum operator S_z with eigenvalue $m\hbar$. ($S_z|m\rangle = m\hbar|m\rangle$).
The expectation value of S_z in the state $|\psi\rangle = \frac{1}{2}|1\rangle + \frac{1}{2}|0\rangle + \frac{1}{\sqrt{2}}|-1\rangle$ is
(a) $\hbar/4$, (b) $-\hbar/2$, (c) $\hbar/2$, (d) $-\hbar/4$.