Training Program for NET in Physics (TPNP)

ASSIGNMENT 4

1. If the cartesian components of $\boldsymbol{\sigma}$ denote the three Pauli matrices, then $\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}$ is equal to
(a) 1 (b) 2 (c) 3 (d) 4

2. If the three components $\sigma_x, \sigma_y, \sigma_z$ of $\boldsymbol{\sigma}$ denote the three Pauli matrices and if $\boldsymbol{A}$ and $\boldsymbol{B}$ denote any two polar vectors, then $(\boldsymbol{\sigma}\cdot\boldsymbol{A})(\boldsymbol{\sigma}\cdot\boldsymbol{B})$ is equal to
(a) $\boldsymbol{A}\cdot\boldsymbol{B}$ (b) $\boldsymbol{\sigma}\cdot(\boldsymbol{A}\times\boldsymbol{B})$ (c) $\boldsymbol{A}\cdot\boldsymbol{B}+i\boldsymbol{\sigma}\cdot(\boldsymbol{A}\times\boldsymbol{B})$ (d) $\boldsymbol{A}\cdot\boldsymbol{B}+\boldsymbol{\sigma}\cdot(\boldsymbol{A}\times\boldsymbol{B})$.

3. If the cartesian components of $\boldsymbol{\sigma}$ denote the three Pauli matrices and if $\hat{\boldsymbol{n}}$ denotes the unit polar vector, then $(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{n}})^2$ is equal to
(a) $\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}$ (b) $\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{n}}$ (c) ± 1 (d) -1

4. If $\boldsymbol{A}$ is a polar vecor, then $(\boldsymbol{\sigma}\cdot\boldsymbol{A})\boldsymbol{\sigma}$ is equal to
(a) $\boldsymbol{\sigma}\times\boldsymbol{\sigma}$ (b) $\boldsymbol{\sigma}\times\boldsymbol{A}$ (c) $\boldsymbol{A}+i\boldsymbol{\sigma}\times\boldsymbol{A}$ (d) $\boldsymbol{A}-i\boldsymbol{\sigma}\times\boldsymbol{A}$

5. If $\boldsymbol{A}$ is a polar vecor, then $\boldsymbol{\sigma}(\boldsymbol{\sigma}\cdot\boldsymbol{A})$ is equal to
(a) $\boldsymbol{\sigma}\times\boldsymbol{\sigma}$ (b) $\boldsymbol{\sigma}\times\boldsymbol{A}$ (c) $\boldsymbol{A}+i\boldsymbol{\sigma}\times\boldsymbol{A}$ (d) $\boldsymbol{A}-i\boldsymbol{\sigma}\times\boldsymbol{A}$

6. Given the Pauli Spin operator $\boldsymbol{\sigma}$ and the orbital angular momentum operator $\boldsymbol{L}$, the expectation value of the operator $\boldsymbol{\sigma}\cdot\boldsymbol{L}$ for an atomic electron in $d_{3/2}$ state is
(a) 3 (b) 1 (c) -1 (d) $-3/2$

7. If $\boldsymbol{\sigma}$ is the Pauli spin operator and $\boldsymbol{A}$ is a polar vector, then $\boldsymbol{\sigma}\cdot\boldsymbol{A}$ can be explicitly written in the following form as a matrix
(a) $\begin{bmatrix} A_z & 0 \\ 0 & -A_z \end{bmatrix}$ (b) $\begin{bmatrix} A_z & A_x \\ A_y & -A_z \end{bmatrix}$ (c) $\begin{bmatrix} A_z & A_x+iA_y \\ A_x-iA_y & -A_z \end{bmatrix}$
(d) $\begin{bmatrix} A_z & A_x-iA_y \\ A_x+iA_y & -A_z \end{bmatrix}$

8. If α denotes the spin-up state and β denotes the spin-down state of an electron, then the matrix element $\langle\beta|\boldsymbol{\sigma}\cdot\boldsymbol{A}|\alpha\rangle$ is given by
(a) A_x+A_y (b) A_x-A_y (c) A_x+iA_y (d) A_x-iA_y

9. If α denotes the spin-up state and β denotes the spin-down state of an electron, then the matrix element $\langle\alpha|\boldsymbol{\sigma}\cdot\boldsymbol{A}|\beta\rangle$ is given by
(a) A_x+A_y (b) A_x-A_y (c) A_x+iA_y (d) A_x-iA_y

10. If α denotes the spin-up state and β denotes the spin-down state of an electron, then the matrix element $\langle\alpha|\boldsymbol{\sigma}\cdot\boldsymbol{A}|\alpha\rangle$ is given by
(a) 1 (b) -1 (c) A_z (d) $-A_z$

11. If α denotes the spin-up state and β denotes the spin-down state of an electron, then the matrix element $\langle\beta|\boldsymbol{\sigma}\cdot\boldsymbol{A}|\beta\rangle$ is given by
(a) 1 (b) -1 (c) A_z (d) $-A_z$

12. For a plane wave e^{ikz} propagating along the z-axis, what is the eigenvalue of the L_z operator?
(a) 0 (b) $1\,\hbar$ (c) $2\,\hbar$ (d) $k\,\hbar$.

13. Using the definition of angular momentum ladder operators $J_\pm = J_x \pm iJ_y$, it can be shown that the commutator $[J_z, J_\pm]_-$ is equal to
(a) 0 (b) $\pm\hbar J_\pm$ (c) $\mp\hbar J_\pm$ (d) $\hbar J_\mp$

14. Using the definition of angular momentum ladder operators $J_\pm = J_x \pm iJ_y$, it can be shown that the commutator $[J_x, J_\pm]_-$ is equal to
(a) 0 (b) $\pm\hbar J_z$ (c) $\mp\hbar J_z$ (d) $-i\hbar J_z$

15. Using the definition of angular momentum ladder operators $J_\pm = J_x \pm iJ_y$, it can be shown that the commutator $[J_y, J_\pm]_-$ is equal to
(a) 0 (b) $\pm\hbar J_z$ (c) $\mp\hbar J_z$ (d) $-i\hbar J_z$

16. Using the definition of angular momentum ladder operators $J_\pm = J_x \pm iJ_y$, it can be shown that the commutator $[J_+, J_-]_-$ is equal to
(a) 0 (b) $2\hbar J_x$ (c) $2\hbar J_y$ (d) $2\hbar J_z$

17. Show that for angular momentum operators, the commutator $[J_x^2, J_z]_-$ is equal to
(a) 0 (b) $[J_y^2, J_z]_-$ (c) $-[J_y^2, J_z]_-$ (d) $-i\hbar J_y$

18. If L_x denotes the x - component of orbital angular momentum operator, then the commutaor $[L_x, y]_-$ is equal to
(a) 0 (b) $i\hbar z$ (c) $i\hbar p_z$ (d) $-i\hbar p_z$

19. If L_x denotes the x - component of orbital angular momentum operator, then the commutaor $[L_x, p_x]_-$ is equal to
(a) 0 (b) $i\hbar p_y$ (c) $i\hbar p_z$ (d) $-i\hbar p_z$

20. The product $(\boldsymbol{\sigma}\cdot\boldsymbol{r})(\boldsymbol{\sigma}\cdot\boldsymbol{L})$ is equal to
(a) 0 (b) $\boldsymbol{r}\cdot\boldsymbol{L}$ (c) $\boldsymbol{\sigma}\cdot\boldsymbol{r}\times\boldsymbol{L}$ (d) $i\boldsymbol{\sigma}\cdot\boldsymbol{r}\times\boldsymbol{L}$