Training Program for NET in Physics (TPNP)

ASSIGNMENT 5

1. If the angular momentum ladder operators J_+ and J_- are defined by

$$J_+ = J_x + iJ_y, \quad J_- = J_x - iJ_y,$$

then J_+J_- is equal to
(a) $J_x^2 + J_y^2$, (b) $\boldsymbol{J}^2 - J_z^2$, (c) $\boldsymbol{J}^2 - J_z(J_z - \hbar)$ (d) $\boldsymbol{J}^2 - J_z(J_z + \hbar)$

2. If the angular momentum ladder operators J_+ and J_- are defined by

$$J_+ = J_x + iJ_y, \quad J_- = J_x - iJ_y,$$

then J_-J_+ is equal to
(a) $J_x^2 + J_y^2$, (b) $\boldsymbol{J}^2 - J_z^2$, (c) $\boldsymbol{J}^2 - J_z(J_z - \hbar)$ (d) $\boldsymbol{J}^2 - J_z(J_z + \hbar)$

3. If the angular momentum ladder operators J_+ and J_- are defined by

$$J_+ = J_x + iJ_y, \quad J_- = J_x - iJ_y,$$

then the matrix element $\langle j'm'|J_+J_-|jm\rangle$ is equal to
(a) $\{j(j+1) - m(m+1)\}\hbar^2\delta_{jj'}\,\delta_{mm'}$ (b) $\{j(j+1) - m(m-1)\}\hbar^2\delta_{jj'}\,\delta_{mm'}$
(c) $\{j(j+1) + m(m+1)\}\hbar^2\delta_{jj'}\,\delta_{mm'}$ (d) $\{j(j+1) + m(m-1)\}\hbar^2\delta_{jj'}\,\delta_{mm'}$

4. If the angular momentum ladder operators J_+ and J_- are defined by

$$J_+ = J_x + iJ_y, \quad J_- = J_x - iJ_y,$$

then the matrix element $\langle j'm'|J_-J_+|jm\rangle$ is equal to
(a) $\{j(j+1) - m(m+1)\}\hbar^2\delta_{jj'}\,\delta_{mm'}$ (b) $\{j(j+1) - m(m-1)\}\hbar^2\delta_{jj'}\,\delta_{mm'}$
(c) $\{j(j+1) + m(m+1)\}\hbar^2\delta_{jj'}\,\delta_{mm'}$ (d) $\{j(j+1) + m(m-1)\}\hbar^2\delta_{jj'}\,\delta_{mm'}$

5. If $\boldsymbol{J}$ ia an angular momentum vector operator and $\boldsymbol{A}$ is any polar vecor, then the matrix element $\langle j, m+1|\boldsymbol{J}\cdot\boldsymbol{A}|j, m\rangle$ is equal to
(a) $\frac{1}{2}(A_x - iA_y)\,\{(j-m)(j+m+1)\}^{1/2}\,\hbar$
(b) $\frac{1}{2}(A_x + iA_y)\,\{(j-m)(j+m+1)\}^{1/2}\,\hbar$
(c) $\frac{1}{2}(A_x - iA_y)\,\{(j+m)(j-m+1)\}^{1/2}\,\hbar$
(d) $\frac{1}{2}(A_x + iA_y)\,\{(j+m)(j-m+1)\}^{1/2}\,\hbar$.

6. If $\boldsymbol{J}$ ia an angular momentum vector operator and $\boldsymbol{A}$ is any polar vecor, then the matrix element $\langle j, m-1|\boldsymbol{J}\cdot\boldsymbol{A}|j,m\rangle$ is equal to
(a) $\frac{1}{2}(A_x - iA_y)\{(j-m)(j+m+1)\}^{1/2}\hbar$
(b) $\frac{1}{2}(A_x + iA_y)\{(j-m)(j+m+1)\}^{1/2}\hbar$
(c) $\frac{1}{2}(A_x - iA_y)\{(j+m)(j-m+1)\}^{1/2}\hbar$
(d) $\frac{1}{2}(A_x + iA_y)\{(j+m)(j-m+1)\}^{1/2}\hbar$.

7. If $\boldsymbol{J}$ ia an angular momentum vector operator and $\boldsymbol{A}$ is any polar vecor, then the matrix element $\langle j, m|\boldsymbol{J}\cdot\boldsymbol{A}|j,m\rangle$ is equal to
(a) 0, (b) $m\hbar$, (c) $A_z m\hbar$, (d) $-A_z m\hbar$.

8. The iso-spin of the deuteron is $T = 0$. If $\eta_p = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ denotes the iso-spin wave function of proton and $\eta_n = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ denotes the iso-spin wave function of neutron, then the iso-spin wave function of the deuteron is given by
(a) $\eta_p(1)\eta_n(2)$ (b) $\eta_n(1)\eta_p(2)$ (c) $\frac{1}{\sqrt{2}}\{\eta_p(1)\eta_n(2) - \eta_n(1)\eta_p(2)\}$
(d) $\frac{1}{\sqrt{2}}\{\eta_p(1)\eta_n(2) + \eta_n(1)\eta_p(2)\}$

9. Consider the deuteron to be a bound state of two nucleons (proton and neutron) in s - state (relative orbital angular momentum $l = 0$) and spin $S = 1$. If $\alpha = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ denotes the spin-up state of neucleon and $\beta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ denotes the spin-down state of nucleon, then the spin wave function of the deuteron is given by
(a) $\alpha(1)\alpha(2)$ (b) $\beta(1)\beta(2)$ (c) $\frac{1}{\sqrt{2}}\{\alpha(1)\beta(2) + \beta(1)\alpha(2)\}$
(d) all the three given in (a), (b) and (c).

10. Pions exist in three charge states, π^+, π^0, π^-. An isospin $T_\pi = 1$ is attributed to the pion and its three charge states are identified with the three projections $m_{T_\pi} = 1, 0, -1$ of the isospin. Nucleon exists in two charge states, proton and neutron and it is attributed an isospin $T_N = 1/2$ with projections $m_{T_N} = +\frac{1}{2}$ denoting the proton and $|m_{T_N} = -\frac{1}{2}$ denoting the neutron. The isospin wave function $|\pi^- p\rangle$ of a system consisting of π^- and proton can be written as
(a) $|T = \frac{3}{2}, m_T = -\frac{1}{2}\rangle$, (b) $|T = \frac{1}{2}, m_T = -\frac{1}{2}\rangle$,
(c) $\frac{1}{2}|T = \frac{3}{2}, m_T = -\frac{1}{2}\rangle + \frac{1}{2}|T = \frac{3}{2}, m_T = -\frac{1}{2}\rangle$,
(d) $\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ -1 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} |T = \frac{3}{2}, m_T = -\frac{1}{2}\rangle + \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ -1 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} |T = \frac{1}{2}, m_T = -\frac{1}{2}\rangle$,
where the symbol $\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ -1 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ denotes the C.G. coefficient.