Physics Olympiad for P.G. students - 2019

Solutions to Multiple Choice Questions

PART A

1. Ans: (c)

$$\begin{aligned} \boldsymbol{\nabla} &= \frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k} \\ \frac{\partial}{\partial x}(x^2+y^2+z^2)^{3/2} &= \frac{3}{2}(x^2+y^2+z^2)^{1/2}2x = 3rx \\ \boldsymbol{\nabla} r^3 &= 3r(x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k}) = 3r\boldsymbol{r}. \end{aligned}$$

2. Ans:(c)
Since $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are mutually orthogonal,

$$\begin{aligned} \boldsymbol{a}\cdot\boldsymbol{c} = 0 &\implies \lambda - 1 + 2\mu = 0. \\ \boldsymbol{b}\cdot\boldsymbol{c} = 0 &\implies 2\lambda + 4 + \mu = 0. \end{aligned}$$

Solving, we get $\lambda = -3,\ \mu = 2$.

3. Ans: (c)
This is a problem in combination. You do not worry about the order in which you choose the 11 players. The number of teams, you can form is ${}^{15}C_{11}$.

$${}^{15}C_{11} = \frac{15!}{11!\,4!} = 1365.$$

4. Ans: (c)

$$E^2 = p^2c^2 + m_0^2c^4 \implies 2EdE = 2pdp\,c^2 \implies v = \frac{dE}{dp} = \frac{p}{E}c^2$$

$$p^2c^2 = E^2 - m_0^2c^4 \implies pc = E\left(1 - \frac{m_0^2c^4}{E^2}\right)^{1/2}$$

$$v = \frac{pc^2}{E} = c\left(1 - \frac{m_0^2c^4}{E^2}\right)^{1/2}.$$

5. Ans: (d)

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\psi = \lambda\,\psi \implies \begin{bmatrix} -\lambda & 1 \\ 1 & -\lambda \end{bmatrix}\psi = 0.$$

For this to have a non-trivial solution,

$$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0 \implies \lambda^2 = 1 \text{ or } \lambda = \pm 1.$$

6. Ans: (b)
$\boldsymbol{r}$ is a vector and $\boldsymbol{\sigma}$ is a pseudo-vector. Hence $\boldsymbol{\sigma} \cdot \boldsymbol{r}$ is a pseudo-scalar.

7. Ans: (b)

$$\oint_c \frac{3z^2 + z}{z^2 - 1} dz = \oint_c \frac{3z^2 + z}{(z+1)(z-1)} dz$$

The integral has simple poles at $z = -1$ and $z = +1$

$$\oint_c \frac{3z^2 + z}{z^2 - 1} dz = 2\pi i \{R_1 + R_2\}$$

$$\begin{aligned} R_1 &= \text{Residue at } z = -1 \\ &= \lim_{z \to -1} \frac{(z+1)(3z^2 + z)}{z^2 - 1} = \lim_{z \to -1} \frac{3z^2 + z}{z - 1} = \frac{3 - 1}{-2} = -1 \\ R_2 &= \text{Residue at } z = +1 \\ &= \lim_{z \to +1} \frac{(z-1)(3z^2 + z)}{z^2 - 1} = \lim_{z \to +1} \frac{3z^2 + z}{z + 1} = \frac{4}{2} = 2 \end{aligned}$$

Hence

$$\oint_c \frac{3z^2 + z}{z^2 - 1} dz = 2\pi i \{-1 + 2\} = 2\pi i.$$

8. Ans: (d)
de Broglie wavelength $\lambda = \frac{h}{p}$.
Since the kinetic energy $E = \frac{p^2}{2m}$, $p = (2mE)^{1/2}$. Therefore $\frac{\lambda_1}{\lambda_2} = \left(\frac{m_2}{m_1}\right)^{1/2}$.

9. Ans: (b)
Since $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$, $L_y = -xp_z + p_x z$.
p_x^2 will commute with everything except x. So, the commutator

$$[p_x^2, L_y]_- = -[p_x^2, x]_- p_z = 2i\hbar p_x p_z,$$

since the commutator

$$[p_x^2, x]_- = p_x [p_x, x]_- + [p_x, x]_- p_x = -2i\hbar p_x.$$

The correct answer is (b).

10. Ans: (a)
For any point outside the charged sphere, it will appear as if the entire charge is concentrated at its centre. So, electrostatic force F between the two spheres is

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2},$$

where $q_1 = q_2 = 6.5 \times 10^{-7}$ Coulomb;
$r = 0.5$ metre;
$1/4\pi\epsilon_0 = 9 \times 10^9$ N m^2 C^{-2}.
Substituting these values, we get F $= 15.21 \times 10^{-3}$ N.

11. Ans: (d)
If N is the number of molecules in volume V, then $V = N/\rho$ and $dV = -(N/\rho^2)d\rho$.
Work done in isothermal compression

$$\begin{aligned} dW &= -\int PdV = \int (a\rho + b\rho^2)(N/\rho^2)d\rho \\ &= N\int_{\rho_0}^{2\rho_0}\left(\frac{a}{\rho} + b\right)d\rho = N\left(a\ln\rho + b\rho\right)|_{\rho_0}^{2\rho_0} \\ &= N\left(a\ln 2 + b\rho_0\right) = \rho_0 V_0\left(a\ln 2 + b\rho_0\right) \end{aligned}$$

12. Ans: (a)

$$x^2 + 4y^2 = 8 \Longrightarrow 2x\frac{dx}{dt} + 8y\frac{dy}{dt} = 0.$$

At $x = 2, y = 1$, $4\frac{dx}{dt} = -8\frac{dy}{dt}$. Since it is given that $\frac{dx}{dt} = 6$, it follows that $\frac{dy}{dt} = -3$.

13. Ans: (a)

$$\begin{aligned} V &= \frac{1}{2}k(t)x^2 \Longrightarrow \langle V\rangle = \frac{1}{2}k(t)\langle x^2\rangle \\ \frac{d}{dt}\langle V\rangle &= \frac{1}{2}\frac{dk}{dt}\langle x^2\rangle + \frac{k}{2}\left\langle x\frac{dx}{dt} + \frac{dx}{dt}x\right\rangle \\ &= \frac{1}{2}\frac{dk}{dt}\langle x^2\rangle + \frac{k}{2m}\langle xp + px\rangle \end{aligned}$$

Alteranative method

Applying Ehrenfest theorem,

$$\frac{d}{dt}\langle A\rangle = \left\langle\frac{\partial A}{\partial t}\right\rangle + \frac{1}{i\hbar}\langle[A, H]\rangle,$$

we get

$$\frac{d}{dt}\langle V\rangle = \left\langle \frac{\partial V}{\partial t}\right\rangle + \frac{1}{i\hbar}\langle [V,H]\rangle = \frac{1}{2}\frac{dk}{dt}\langle x^2\rangle + \frac{1}{i\hbar}\left\langle \left[V, \frac{p^2}{2m}+V\right]\right\rangle,$$

Let us now evaluate the commutator

$$\begin{aligned}[V,H] &= \frac{1}{2}kx^2\left(\frac{p^2}{2m}+\frac{1}{2}kx^2\right) - \left(\frac{p^2}{2m}+\frac{1}{2}kx^2\right)\frac{1}{2}kx^2 = \frac{1}{2}\frac{k}{2m}[x^2,p^2] \\ &= \frac{k}{4m}\left(x[x,p^2]+[x,p^2]x\right) = \frac{k}{4m}2i\hbar(xp+px)\end{aligned}$$

since $[x,p^2] = 2i\hbar p$. Hence

$$\frac{d}{dt}\langle V\rangle = \left\langle \frac{\partial V}{\partial t}\right\rangle + \frac{1}{i\hbar}\langle [V,H]\rangle = \frac{1}{2}\frac{dk}{dt}\langle x^2\rangle + \frac{k}{2m}\langle xp+px\rangle.$$

14. Ans: (b)
Due to parity conservation in strong interaction, the two neutrons in the final state should be in p state ($l = 1$). The total wave function of two neutrons should be antisymmetric.

$$\begin{aligned}\text{Total wave function (n,n)} &= \text{(orbital) (spin) (isospin)} \\ &= \text{(antisymmetric) (symmetric) (symmetric)}\end{aligned}$$

Hence spin of n-n: $S = 1$ (Symmetric)

15. Ans: (b)
Radius R is proportional to $A^{1/3}$.

$$\frac{R_{\text{Al}}}{R_{\text{Zn}}} = \frac{(27)^{1/3}}{(64)^{1/3}} = \frac{3}{4}.$$

So, $R_{\text{Al}} = \frac{3}{4}\times 4.8\times 10^{-13} = 3.6\times 10^{-13}$ cm.

16. Ans: (c)
Nucleon configuration of the ground state and the first excited state of $^{17}O_8$:
Ground state: $\left(1s_{1/2}\right)^{2p,2n}\left(1p_{3/2}\right)^{4p,4n}\left(1p_{1/2}\right)^{2p,2n}\left(1d_{5/2}\right)^{1n}$
First excited state: $\left(1s_{1/2}\right)^{2p,2n}\left(1p_{3/2}\right)^{4p,4n}\left(1p_{1/2}\right)^{2p,2n}\left(2s_{1/2}\right)^{1n}$
Spin-parity of Ground state of $^{17}O_8 = 5/2^+$.
Spin-parity of First excited state of $^{17}O_8 = 1/2^+$.

17. Ans: (a)
The rotational energy levels of the nucleus

$$E_j = \frac{\hbar^2}{2\mathcal{I}} j(j+1),$$

where $\mathcal{I}$ denotes the moment of inertia and j can take values $0, 2, 4, \cdots$. Since $j = 0$ corresponds to the ground state, $j = 2$ corresponds to the first excited state and $j = 4$ corresponds to the second excited state.

$$E_2 = \frac{6\hbar^2}{2\mathcal{I}} = 90\,\text{KeV}; \qquad E_4 = \frac{20\hbar^2}{2\mathcal{I}} = 300\,\text{KeV}$$

18. Ans: (c)

$$\begin{aligned} p_x &= \frac{\partial L}{\partial \dot{x}} = 2a\dot{x}; \qquad\qquad p_y = \frac{\partial L}{\partial \dot{y}} = 2b\dot{y} \\ H &= \sum_i p_i \dot{q}_i - L \;=\; p_x\dot{x} + p_y\dot{y} - a\dot{x}^2 - b\dot{y}^2 + kxy \\ &= 2a\dot{x}^2 + 2b\dot{y}^2 - a\dot{x}^2 - b\dot{y}^2 + kxy \;=\; a\dot{x}^2 + b\dot{y}^2 + kxy \\ &= \frac{p_x^2}{4a} + \frac{p_y^2}{4b} + kxy \end{aligned}$$

19. Ans: (b)
$t'_B - t'_A = \gamma \left\{ (t_B - t_A) - \frac{v}{c^2}(x_B - x_A) \right\}.$
Data given: $t_B - t_A = 0$; $x_B - x_A = 10 \times 10^3$ m; $v/c = 0.95$;
$c = 3 \times 10^8$ m.
$\gamma = \frac{1}{\sqrt{1 - (v^2/c^2)}} = 3.2026.$
Substituting these values, we get $t'_B - t'_A = -1.0142 \times 10^{-4}$ sec.
From the negative sign, we infer that the flash at B occurs first and the flash at A happens after a time interval 1.0142×10^{-4} sec.

20. Ans: (d)
NaCl belongs to f.c.c lattice and so there are $n = 4$ molecules per unit cell.

$$a = \left(\frac{nM}{N_a\,\rho}\right)^{1/3} = \left(\frac{4 \times 58.45}{6 \times 10^{26} \times 2170}\right)^{1/3} = 5.64 \times 10^{-10}\,\text{m}$$

21. Ans: (c)

22. Ans: (a)
Arsenic, Antimony and Phosphorous are pentavalent impurities and Aluminium is a trivalent impurity. Hence the intrinsic semiconductor doped with Aluminium is a p-type semiconductor.

23. Ans: (c)

24. Ans: (b)

$$\boldsymbol{B} = \text{Curl}\,\boldsymbol{A} = \boldsymbol{\nabla} \times \boldsymbol{A} = \begin{bmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix}$$

$$\boldsymbol{B} = B_0\hat{\boldsymbol{k}} = \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\boldsymbol{k}}$$

Let $A_y = 0$. Then $B_0 dy = -\partial A_x$ or $B_0 y = -A_x$.
This leads to a possible choice of the vector potential $\boldsymbol{A} = -B_0 y\hat{\boldsymbol{i}}$.

25. Ans: (c)
Units of $h = JS,\ c = m/S,\ E = J$. Unit of cross section is m^2.

26. Ans: (b)
Total effective resistance in the circuit = 50+70 = 120 Ω.
Current through 50 Ω resistance = $\frac{12}{120} = 0.1$ A.
Voltage across the resistance 50 Ω = 50 × 0.1 = 5 Volts.

27. Ans: (a)

$$\begin{aligned} \overline{\left(A+\bar{B}\right)\cdot C} &= \overline{\left(A+\bar{B}\right)} + \bar{C}, && \text{using De Morgan's theorem} \\ &= (\bar{A}\cdot\bar{\bar{B}}) + \bar{C}, && \text{using De Morgan's theorem} \\ &= (\bar{A}\cdot B) + \bar{C} \end{aligned}$$

28. Ans: (c)
The total wave function of two protons should be antisymmetric. Since the spin part is symmetric, the orbital part should be antisymmetric.

29. Ans: (b)
It is a decay by weak interaction, for which $\Delta S = 1, 0$ are allowed values. In (b) $\Delta S = 2$, since $S = -3$ for Ω^- and $S = -1$ for Λ^0.

30. Ans: (c)

PART B

31. Ans: (d)

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x}-1}{e^{2x}+1} = \frac{1}{y}$$

$$\left(e^{2x}-1\right) y = e^{2x}+1 \Longrightarrow e^{2x}(y-1) = y+1 \Longrightarrow e^{2x} = \frac{y+1}{y-1}$$

$$e^x = \sqrt{\frac{y+1}{y-1}} \Longrightarrow x = \ln\sqrt{\frac{y+1}{y-1}}.$$

32. Ans: (a)
In the stationary lift

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

When the lift accelerates upwards with an acceleration $g/4$,the effective acceleration on the pendulum is $g' = g + g/4 = 5g/4$.

$$T' = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{4}{5}\frac{l}{g}} = \frac{2T}{\sqrt{5}}$$

33. Ans: (a)

$$F(\boldsymbol{k}) = \frac{1}{(2\pi)^{3/2}}\int_{-\infty}^{\infty} e^{-\{(x^2/a^2)+ik_xx\}}dx\int_{-\infty}^{\infty} e^{-\{(y^2/a^2)+ik_yy\}}dy\int_{-\infty}^{\infty} e^{-\{(z^2/a^2)+ik_zz\}}dz$$

$$\int_{-\infty}^{\infty} e^{-\{(x^2/a^2)+ik_xx\}}dx = \int_{-\infty}^{\infty} e^{-\left(\frac{x}{a}+i\frac{k_xa}{2}\right)^2} e^{-k_x^2a^2/4}dx$$

$$= e^{-k_x^2a^2/4}\int_{-\infty}^{\infty} e^{-\left(\frac{x}{a}+i\frac{k_xa}{2}\right)^2}dx$$

Put $y = \frac{x}{a} + \frac{ik_xa}{2}$. Then $dy = \frac{1}{a}dx$.

$$\int_{-\infty}^{\infty} e^{-\left(\frac{x}{a}+i\frac{k_xa}{2}\right)^2}dx = a\int_{-\infty}^{\infty} e^{-y^2}dy = a\sqrt{\pi}$$

Similarly performing the other integrals and substituting these values, we get

$$F(\boldsymbol{k}) = \frac{1}{(2\pi)^{3/2}}a^3(\pi)^{3/2}e^{-k_x^2a^2/4} = \left(\frac{a}{\sqrt{2}}\right)^3 e^{-k^2a^2/4}.$$

34. Ans: (b)

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$$
$$(D^2 - 3D + 2)x = 0, \quad \text{where} \quad D = \frac{d}{dt}.$$

If m are the roots, then

$$m^2 - 3m + 2 = 0 \Longrightarrow (m-1)(m-2) = 0.$$

This yields the values $m = 1$ and $m = 2$ and the solution is

$$x(t) = Ae^t + Be^{2t},$$

where A and B are constants to be determined by the boundary conditions $x = 0$ at $t = 0$ and $x = 1$ at $t = 1$.

$$A + B = 0; \qquad Ae + Be^2 = 1.$$

This yields the values $A = -B$ and $A(e - e^2) = 1$ or $A = -B = \frac{1}{e-e^2}$

At $t = 2$, $\qquad x = Ae^t + Be^{2t} = \dfrac{1}{e - e^2}(e^2 - e^4) = e + e^2.$

35. Ans: (d)
Condition for the existence of a non-trivial solution:

$$\det \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & b & 2c \end{bmatrix} = 0 \quad \Longrightarrow \quad (4c - 3b) - (2c - 6) + (b - 4) = 0.$$

This yields the condition $b = c + 1$. Substituting this, we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & b & 2b-2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This yields the following set of equations:

$$\begin{aligned} x + y + z &= 0 \\ x + 2y + 3z &= 0 \\ 2x + by + (2b-2)z &= 0 \end{aligned}$$

This yields the values: $x = z,\ y = -2x$. Let $x = 1$, then $y = -2,\ z = 1$. This yields the un-normalized solution

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}.$$

The normalized solution is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}.$$

36. Ans: (b)
For monoatomic gas, $\gamma = 5/3$ and for adiabatic process, $TV^{\gamma-1}$ = constant.

$$T_1(L_1A)^{2/3} = T_2(L_2A)^{2/3} \quad \Longrightarrow \quad \frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{2/3}.$$

37. Ans: (c)
The arrangement shown in figure is equivalent to two capacitors of capacitances C_1 and C_2 connected in series.

$$C_1 = \frac{K_1\epsilon_0 A}{d/2}; \qquad C_2 = \frac{K_2\epsilon_0 A}{d/2}.$$

If C is the net capacitance, then

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1\epsilon_0 A} + \frac{d}{2K_2\epsilon_0 A} = \frac{d}{2\epsilon_0 A}\left(\frac{K_1 + K_2}{K_1K_2}\right).$$
$$C = \frac{2\epsilon_0 A}{d}\left(\frac{K_1K_2}{K_1 + K_2}\right).$$

38. Ans: (c):
Kinetic energy of a particle: $K = \frac{1}{2}mv^2$. Its velocity is $v = \sqrt{\frac{2K}{m}}$.
If the particle carries a charge q and moves in a uniform magnetic field B, then it will describe a circle of radius R.

$$\frac{mv^2}{R} = Bqv \quad \Longrightarrow \quad \frac{mv}{R} = Bq.$$

This yields the relation for the velocities and kinetic energies of alpha particle and proton describing the circle of the same radius R, in the same magnetic field B.

$$\frac{m_p v_p}{R} = Bq_p; \ \frac{m_\alpha v_\alpha}{R} = Bq_\alpha \quad \Longrightarrow \quad \frac{v_\alpha}{v_p} = \frac{q_\alpha m_p}{q_p m_\alpha} = \frac{2}{4} = \frac{1}{2}$$

Since $q_\alpha = 2q_p$ and $m_\alpha = 4m_p$, we obtain $\frac{v_\alpha}{v_p} = \frac{1}{2}$. In terms of kinetic energies,

$$\frac{v_\alpha}{v_p} = \left(\frac{2K_\alpha}{m_\alpha}\frac{m_p}{2K_p}\right)^{1/2} = \frac{1}{2}\left(\frac{K_\alpha}{K_p}\right)^{1/2}.$$

Since $\frac{v_\alpha}{v_p} = \frac{1}{2}$, it follows that $K_\alpha = K_p = 1$ MeV.

39. Ans: (d)
The speed of electromagnetic wave in a medium of relative permittivity ϵ_r and relative permeability μ_r is given by

$$v = \frac{1}{\sqrt{\epsilon_r\epsilon_0\mu_r\mu_0}} = \frac{c}{\sqrt{\epsilon_r\mu_r}}, \quad \text{since } c = \frac{1}{\sqrt{\epsilon_0\mu_0}}.$$

Substituting the values $\epsilon_r = 2.14,\ \mu_r = 1.3,\ c = 3 \times 10^8$, we get

$$v = \frac{3 \times 10^8}{\sqrt{2.14 \times 1.3}} = 1.8 \times 10^8\,\mathrm{m\,s^{-1}}.$$

40. Ans: (b)
Fringe width $\beta = \frac{\lambda D}{d}$. Substituting the values $\lambda = 6000 \times 10^{-10}$ m, D= 2 m, $d = 4 \times 10^{-3}$ m, we get $\beta = 0.3$ mm

41. Ans: (b)
The decay constant (λ) is the reciprocal of mean life (τ).

$$\lambda_\alpha = \frac{1}{1620};\ \lambda_\beta = \frac{1}{405};\ \lambda = \lambda_\alpha + \lambda_\beta.$$

λ is the total decay constant.

$$\lambda = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}\text{ per year.}$$

Applying radioactive decay formula,

$$N = N_0 e^{-\lambda t},$$

we can find the time taken for the three-fourth of the sample to decay or one-fourth of the sample remaining. $N = N_0/4$.

$$\frac{N}{N_0} = \frac{1}{4} = e^{-\lambda t} \Longrightarrow \lambda t = \ln 4 \Longrightarrow t = \frac{\ln 4}{\lambda} = \frac{2\ln 2}{\lambda}$$

Substituting the values, we get $t = 2 \times 324 \times 0.693 = 449$ years.

42. **Ans: (c)**

$V(x)$ is minimum at the value of x at which $\frac{dV}{dx} = 0$.

$$V(x) = -\frac{1}{2}kx^2 + \frac{1}{4}\lambda x^4 \Longrightarrow \frac{dV}{dx} = -kx + \lambda x^3 = 0 \Longrightarrow x^2 = \frac{k}{\lambda}.$$

$V(x)$ is minimum at $x = x_0$: $x_0^2 = k/\lambda$ and it corresponds to the equilibrium point.

Expanding $V(x)$ around the point $x = x_0$, we get

$$\begin{aligned} V(x) &= V(x_0) + (x - x_0)\left.\frac{dV}{dx}\right|_{x=x_0} + \frac{(x-x_0)^2}{2!}\left.\frac{d^2V}{dx^2}\right|_{x=x_0} + \cdots \\ V(x) - V(x_0) &= \frac{(x-x_0)^2}{2}\left.\frac{d^2V}{dx^2}\right|_{x=x_0}, \quad \text{since } \left.\frac{dV}{dx}\right|_{x=x_0} = 0. \end{aligned}$$

$$\frac{d^2V}{dx^2} = -k + 3\lambda x^2; \qquad \left.\frac{d^2V}{dx^2}\right|_{x=x_0} = -k + 3\lambda\frac{k}{\lambda} = 2k.$$

This yields

$$V(x) - V(x_0) = \frac{(x-x_0)^2}{2}(2k) = \frac{1}{2}K(x-x_0)^2,$$

where $K = 2k$. This represents a simple harmonic motion about the point $x = x_0$ with angular frequency $\omega = \sqrt{K/m} = \sqrt{\frac{2k}{m}}$ since $K = 2k$. If ν is the frequency of oscillations, then

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

43. Ans: (a)

$$\begin{aligned} \boldsymbol{J}\cdot\boldsymbol{A} &= J_xA_x + J_yA_y + J_zA_z \\ &= \frac{1}{2}(J_+ + J_-)A_x + \frac{1}{2i}(J_+ - J_-)A_y + J_zA_z \\ &= \frac{1}{2}J_+(A_x - iA_y) + \frac{1}{2}J_-(A_x + iA_y) + J_zA_z \end{aligned}$$

Since the operator J_+ alone will connect the initial state $|j,m\rangle$ to the final state $|j,m+1\rangle$ yielding the matrix element

$$\langle j, m+1|J_+|j,m\rangle = \{(j-m)(j+m+1)\}^{1/2}\hbar,$$

it follows that

$$\langle j, m+1|\boldsymbol{J}\cdot\boldsymbol{A}|j,m\rangle = \tfrac{1}{2}(A_x - iA_y)\{(j-m)(j+m+1)\}^{1/2}\hbar.$$

So, the correct answer is: (a).

44. Ans: (a)

The particle is in a mixture of states with quantum numbers $n = 2$ and $n = 4$, since the normalized wave function and energy of a particle in a definite state n in an infinite potential well of width L is given by

$$\psi_n = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L}. \qquad E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{n^2h^2}{8mL^2} \text{ (Note: } \hbar = h/(2\pi))$$

The given normalized wave function of the particle is

$$\psi(x) = \sqrt{\frac{2}{L}}\left\{\frac{3}{5}\sin\frac{2\pi x}{L} + \frac{4}{5}\sin\frac{4\pi x}{L}\right\} = \frac{3}{5}\psi_2 + \frac{4}{5}\psi_4.$$

The particle is in a mixture of states with $n = 2$ and $n = 4$ with probabilities $\left(\frac{3}{5}\right)^2$ and $\left(\frac{4}{5}\right)^2$.Thus the energy of the particle is

$$E = \frac{9}{25}E_2 + \frac{16}{25}E_4 = \frac{9}{25}\left(\frac{h^2}{2mL^2}\right) + \frac{16}{25}\left(\frac{2h^2}{mL^2}\right) = \frac{73}{50}\left(\frac{h^2}{mL^2}\right).$$

45. Ans: (a)

The force between two long parallel wires, each of length ℓ, carrying currents I_1 and I_2 in the same direction but separated by a distance r is given by Eq.

$$F = \frac{\mu_0 I_1 I_2 \ell}{2\pi r}.$$

This is a force of attraction.

If three long parallel conductors A,B,C carrying current in the same direction as shown in figure, then the middle conductor B experiences an attractive force due to conductors A and C, placed on either side. If F_1 and F_2 represent respectively the two forces that the conductor C experiences due to conductors A and C on either side, the net force acting on C is

$$F = F_2 - F_1 = \frac{\mu_0 I_B I_C \ell}{2\pi r_{BC}} - \frac{\mu_0 I_a I_B \ell}{2\pi r_{AB}} = \frac{\mu_0 \ell}{2\pi}\left\{\frac{I_B I_C}{r_{BC}} - \frac{I_A I_B}{r_{AB}}\right\}.$$

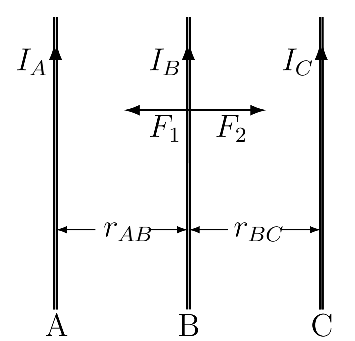

Fig.

Substituting the values, $I_A = 2$ Amp, $I_B = 3$ Amp, I_C =4 Amp, $\ell = 10$ metres, $r_{AB} = r_{BC} = 10$ cm =0.1 m and $\mu_0 = 4\pi \times 10^{-7}$ henry/metre, we get

$$F = \frac{2 \times 10^{-7} \times 10}{0.1}(12 - 6) = 1200 \times 10^{-7} = 1.2 \times 10^{-4} N$$

46 . Ans: (b)

According to Gauss' law in electrostatics, the total normal electric flux ϕ over a closed surface depends on the total charge q residing within the closed surface.

$$\phi = \frac{q}{\epsilon_0},$$

where ϵ_0 is the permittivity of the medium. Let q be the charge residing within a sphere of radius $R/2$ and Q be the charge residing in the whole solid sphere of radius R.

$$\begin{aligned} q &= 4\pi \int_0^{R/2} \rho(r) r^2 dr = 4\pi\rho_0 \int_0^{R/2} \left(1 - \frac{ar}{R}\right) r^2 dr \\ &= 4\pi\rho_0 \left| \frac{r^3}{3} - \frac{a}{R}\frac{r^4}{4} \right|_0^{R/2} = 4\pi\rho_0 R^3 \left(\frac{1}{24} - \frac{a}{64}\right) \\ &= 4\pi\epsilon_0 \frac{R^2}{4} E_{R/2}, \quad \text{by Gauss' law} \end{aligned}$$

So

$$E_{R/2} = \frac{\rho_0}{\epsilon_0} 4R \left(\frac{1}{24} - \frac{a}{64}\right)$$

$$\begin{aligned} Q &= 4\pi \int_0^{R} \rho(r) r^2 dr = 4\pi\rho_0 \int_0^{R} \left(1 - \frac{ar}{R}\right) r^2 dr \\ &= 4\pi\rho_0 \left| \frac{r^3}{3} - \frac{a}{R}\frac{r^4}{4} \right|_0^{R} = 4\pi\rho_0 R^3 \left(\frac{1}{3} - \frac{a}{4}\right) \\ &= 4\pi\epsilon_0 R^2 E_R, \quad \text{by Gauss' law} \end{aligned}$$

This yields

$$E_R = \frac{\rho_0}{\epsilon_0} R \left(\frac{1}{3} - \frac{a}{4}\right)$$

It is given that $E_{R/2} = \frac{5}{4} E_R$.. Hence

$$4\left(\frac{1}{24} - \frac{a}{64}\right) = \frac{5}{4}\left(\frac{1}{3} - \frac{a}{4}\right)$$

This yields the value $a = 1$.

47. Ans: (a):
The magnetic induction along the axis of the solenoid is given by

$$B = \mu_0 n I,$$

where n denotes the number of turns per unit length and I the current passing through the solenoid. Substituting the values $n = 1000/2 = 500,\ I = 5$ Amps and $\mu_0 = 4\pi \times 10^{-7}$ henry/metre, we get

$$B = 3.14 \times 10^{-3} \text{ Tesla.}$$

48. Ans: (c):
Let us consider the collision of two particles a and b resulting in a final state c of an aggregate of particles. $a + b \to c$.
In the present case, the four-momentum of the final state $\mathbf{P}_c$ in c.m. system for the threshold production of $\Lambda\overline{\Lambda}$ is

$$\mathbf{P}_c = (2M_\Lambda,\ 0).$$

The conservation of four-momentum leads to the relation

$$\begin{aligned}(\mathbf{P}_a + \mathbf{P}_b)^2 &= \mathbf{P}_c^2 \\ \mathbf{P}_a^2 + \mathbf{P}_b^2 + 2\mathbf{P}_a \cdot \mathbf{P}_b &= \mathbf{P}_c^2 \\ M_p^2 + M_p^2 + 2\mathbf{P}_a \cdot \mathbf{P}_b &= (2M_\Lambda)^2 \\ 2\mathbf{P}_a \cdot \mathbf{P}_b &= 4M_\Lambda^2 - 2M_p^2.\end{aligned}$$

In the laboratory system, the target is at rest. So,

$$\mathbf{P}_a = (E_a,\ \boldsymbol{p}_a); \qquad \mathbf{P}_b = (M_p,\ 0).$$

Substituting these values of four-momenta, we get

$$2\mathbf{P}_a \cdot \mathbf{P}_b = 2E_a M_p = 4M_\Lambda^2 - 2M_p^2.$$

Thus, we find the total energy of the incident proton required for threshold production of $\Lambda\overline{\Lambda}$ pair production to be

$$E_a = \frac{1}{M_p}(2M_\Lambda^2 - M_p^2).$$

Substituting the values $M_p = 938$ MeV and $M_\Lambda = 1115$ MeV, we get $E_a =$ 1712.8 MeV. This is the total energy including the rest mass energy of the incident proton. So the required kinetic energy of the incident proton in laboratory frame for $\Lambda\overline{\Lambda}$ pair production is

$$T_p = 1712.8 - 938 = 774.8\,\text{MeV}.$$

49. Ans: (a):
Since the electrons in the betatron are accelerated to very high velocities, it is reasonable to assume $v \approx c$. At such high velocities, it is justifiable to neglect the rest mass energy of the electron and assume that the momentum of electron $p = mv \approx E/c$. Under these approximations

$$\frac{mv^2}{R} = Bev; \quad mv = RBe; \quad \frac{E}{c} = mv = RBe.$$

Thus, we obtain the energy of the emerging electron to be

$$E = RBec,$$

50. Ans: (b)
First, let us derive an expression for the Fermi energy.
Number of quantum states available for electrons in momentum range p and $p + dp$ in volume V is $V 4\pi p^2 dp/h^3$. Since 2 electrons, one with spin up and another with spin down, can occupy a quantum state, the number of electrons in volume V in momentum range 0 to p_f is

$$N = 2\int_0^{p_F} \frac{V 4\pi p^2 dp}{h^3} = \frac{8\pi V p_F^3}{3h^3}.$$

If E_f is the Fermi energy, then $p_f = (2mE_F)^{1/2}$.

$$N = \frac{8\pi V(2mE_F)^{3/2}}{3h^3} \Longrightarrow E_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}.$$

Volume of unit cell $= a^3 = \left(5.34 \times 10^{-10}\right)^3$ m^3. For bcc solid, there are 2 atoms per unit cell. So, the number of atoms per unit volume (m^3) is $2/a^3$. Since it is given that it is a monovalent solid, the number of valence electrons n is the same as the number of atoms. So, the number of valence electrons per unit volume is

$$\begin{aligned} \frac{N}{V} &= n = \frac{2}{a^3}. \\ E_F &= \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{2/3} = \frac{h^2}{2m}\left(\frac{3}{4\pi a^3}\right)^{2/3} = \frac{h^2}{2ma^2}\left(\frac{3}{4\pi}\right)^{2/3} \end{aligned}$$

Substituting the values $h = 6.626 \times 10^{-34}$ Js, $m = 9.109 \times 10^{-31}$ kg and $a = 5.34 \times 10^{-10}$ m, we get E_F in Joules. To get E_F in eV, we need to divide it by 1.602×10^{-19} since 1 eV $= 1.602 \times 10^{-19}$ J.

$$E_F = \left(\frac{1}{1.602 \times 10^{-19}}\right) \frac{6.626^2 \times 10^{-68}}{2 \times 9.11 \times 10^{-31} \times 5.34^2 \times 10^{-20}} \left(\frac{3}{4\pi}\right)^{2/3} = 2.03\,\text{eV}.$$

51. Ans: (d)
Let us use the concept of phase-space. The number of quantum states available for particles with momentum lying between p and $p + dp$ and confined within a volume V is:

$$dN = \frac{V 4\pi p^2 dp}{h^3},$$

where h^3 is the volume occupied by one quantum state in phase-space due to uncertainty principle, h denoting the Planck's constant. If m is the mass of the particle, then its kinetic energy $\epsilon = p^2/(2m)$

$$p^2 = 2m\epsilon \Longrightarrow pdp = md\epsilon; \quad p^2 dp = (2m\epsilon)^{1/2} m\, d\epsilon = \sqrt{2}\, m^{3/2} \epsilon^{1/2} d\epsilon.$$

Since 2 electrons, one with spin up and another with spin down, can occupy each quantum state, the total number of electrons in volume V is

$$N = \frac{8\pi V}{h^3} \sqrt{2}\, m^{3/2} \int_0^{\epsilon_F} \epsilon^{1/2} d\epsilon = \frac{8\pi V}{3h^3} (2m\epsilon_F)^{3/2}.$$

The total energy of electrons in volume V is

$$E = \frac{8\pi V}{h^3} \sqrt{2}\, m^{3/2} \int_0^{\epsilon_F} \epsilon^{3/2} d\epsilon = \frac{8\pi V}{5h^3} (2m)^{3/2} (\epsilon_F)^{5/2}.$$

The average energy of an electron is

$$\bar{\epsilon} = \frac{E}{N} = \frac{3}{5}\, \epsilon_F.$$

52. Ans: (d)
The radial function is normalized if the probability of finding the electron is 1 in the entire range of radial distance lying between $r = 0$ and $r = \infty$.

$$\int_0^\infty |R_{21}(r)|^2 r^2 dr = \frac{1}{24a^5} \int_0^\infty r^4 e^{-r/a} dr = \frac{1}{24a^5} \frac{4!}{(1/a)^5} = 1.$$

Probability of finding the electron at the radial distance r is

$$P_r = \frac{1}{24a^5} r^4 e^{-r/a}$$

At maximum probability, $\frac{dp_r}{dr} = 0$.

$$\frac{dP_r}{dr} = \frac{1}{24a^5} \frac{d}{dr} \left(r^4 e^{-r/a} \right) = \frac{1}{24a^5} \left(4r^3 e^{-r/a} a - \frac{1}{a} r^4 e^{-r/a} \right) = 0$$

This yields the value $r = 4a$ for maximum probability. So, $r_0 = 4a$.

53. Ans: (a)

Let us use the following definite integrals:

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}; \qquad \int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}.$$

The Hamiltonian, trial wave function and the energy are as follows:

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x); \qquad \psi(x) = Ae^{-bx^2}; \qquad E = \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}.$$

We have

$$\begin{aligned}
\langle\psi|\psi\rangle &= A^2 \int_{-\infty}^{\infty} e^{-2bx^2} dx = A^2\sqrt{\frac{\pi}{2b}} \\
\langle\psi|H|\psi\rangle &= \left\langle \psi \left| -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x) \right| \psi \right\rangle \\
&= I_1 + I_2 \\
\frac{d\psi}{dx} &= A\frac{d}{dx}e^{-bx^2} = Ae^{-bx^2}(-2bx) \\
\frac{d^2\psi}{dx^2} &= -2Ab\frac{d}{dx}\left(xe^{-bx^2}\right) = -2Ab\left\{e^{-bx^2} + xe^{-bx^2}(-2bx)\right\} \\
&= -2Abe^{-bx^2}(1-2bx^2) \\
I_1 = \left\langle \psi \left| -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} \right| \psi \right\rangle &= A^2\frac{\hbar^2}{2m}\int_{-\infty}^{\infty}\left\{2be^{-2bx^2}(1-2bx^2)\right\} dx \\
&= \frac{A^2\hbar^2}{2m}2b\left\{\int_{-\infty}^{\infty} e^{-2bx^2} dx - 2b\int_{-\infty}^{\infty} x^2 e^{-2bx^2} dx\right\} \\
&= \frac{A^2\hbar^2 b}{m}\left\{\sqrt{\frac{\pi}{2b}} - 2b\frac{1}{4b}\sqrt{\frac{\pi}{2b}}\right\} \\
&= \frac{A^2\hbar^2}{2m}\sqrt{\frac{\pi b}{2}} = \frac{A^2\hbar^2}{4m}\sqrt{2\pi b}. \\
I_2 = -\alpha\langle\psi|\delta(x)|\psi\rangle &= -A^2\alpha\int_{-\infty}^{\infty} e^{-2bx^2}\delta(x)dx \\
&= -A^2\alpha \\
E = \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} &= \frac{A^2\left(\frac{\hbar^2}{4m}\sqrt{2\pi b} - \alpha\right)}{A^2\sqrt{\frac{\pi}{2b}}} = \sqrt{\frac{2b}{\pi}}\left(\frac{\hbar^2}{4m}\sqrt{2\pi b} - \alpha\right) \\
&= \frac{\hbar^2}{4m}2b - \sqrt{\frac{2b}{\pi}}\alpha.
\end{aligned}$$

To find the value of the parameter b for which the energy is minimum, put

$\frac{dE}{db} = 0$. The resulting value of b is obtained as

$$\sqrt{2\pi b} = \frac{2m\alpha}{\hbar^2}.$$

Substituting this value of b in the expression for E, we get

$$E = -\frac{m\alpha^2}{\pi\hbar^2}.$$

54. Ans: (c)

Note that $\int_{-\infty}^{\infty} e^{-\alpha^2x^2} x\,dx = \int_{-\infty}^{\infty} e^{-\alpha^2x^2} x^3\,dx = 0; \qquad \int_{-\infty}^{\infty} e^{-\alpha^2x^2} x^2 dx = \frac{\sqrt{\pi}}{2\alpha^3}$

$$\begin{aligned}\langle 1|x|0\rangle = \langle 0|x|1\rangle &= \left(\frac{2}{\pi}\right)^{1/2} \alpha^2 \int_{-\infty}^{\infty} e^{-\alpha^2x^2} x^2 dx = \left(\frac{2}{\pi}\right)^{1/2} \alpha^2 \left(\frac{\sqrt{\pi}}{2\alpha^3}\right) \\ &= \frac{1}{\sqrt{2}\,\alpha} = \left(\frac{\hbar}{2m\omega}\right)^{1/2}.\end{aligned}$$

$\langle\psi|x|\psi\rangle = c_0^2\langle 0|x|0\rangle + c_1^2\langle 1|x|1\rangle + 2c_0c_1\langle 0|x|1\rangle = 0 + 0 + 2c_0c_1\langle 0|x|1\rangle = \langle 0|x|1\rangle$, if $c_0 = c_1 = \frac{1}{\sqrt{2}}$ such that $c_0^2 + c_1^2 = 1$.

55. Ans: (b)

Let us write down the four-momentum equation for the K^--decay
$K^- \to \mu^- + \overline{\nu}_\mu$.

$$\mathbf{P}_K = \mathbf{P}_\mu + \mathbf{P}_\nu \quad \text{or} \quad \mathbf{P}_K - \mathbf{P}_\nu = \mathbf{P}_\mu.$$

Squaring the latter form of the four-momentum equation, we get, in the rest frame of K^-,

$$\begin{aligned}\mathbf{P}_K^2 + \mathbf{P}_\nu^2 - 2\mathbf{P}_K \cdot \mathbf{P}_\nu &= \mathbf{P}_\mu^2 \\ m_K^2 + 0 - 2m_K E_\nu &= m_\mu^2 \\ E_\nu &= \frac{m_K^2 - m_\mu^2}{2m_K}.\end{aligned}$$

In deducing the above equation, we have used the relation that the four-momentum square is just equal to the rest mass square of the particle. Also, the scalar product of two four-momenta $\mathbf{P}_1 \cdot \mathbf{P}_2 = E_1E_2 - \boldsymbol{p}_1 \cdot \boldsymbol{p}_2$.. If the momentum of one of the particles is zero, the scalar product of three-momenta of the two particles becomes zero. Substituting the masses n energy units $m_K = 494$ MeV, $m_\mu = 106$ MeV, we get $E_\nu = 235.63$ MeV. So, the correct answer is (b).

56. Ans: (c)

The total spin $\boldsymbol{S} = \boldsymbol{S}_e + \boldsymbol{S}_p$ of electron - proton system can be either 1 0r 0.

$$\begin{aligned} \boldsymbol{S}^2 = (\boldsymbol{S}_e + \boldsymbol{S}_p)^2 &= \boldsymbol{S}_e^2 + \boldsymbol{S}_p^2 + 2\,\boldsymbol{S}_e \cdot \boldsymbol{S}_p \\ \langle \boldsymbol{S}_e \cdot \boldsymbol{S}_p \rangle &= \frac{1}{2}\left\{ \langle \boldsymbol{S}^2 \rangle - \langle \boldsymbol{S}_e^2 \rangle - \langle \boldsymbol{S}_p^2 \rangle \right\} \\ &= \frac{1}{2}\left\{ S(S+1) - \frac{3}{2} - \frac{3}{2} \right\} \hbar^2 \end{aligned}$$

For S=1, $\langle \boldsymbol{S}_e \cdot \boldsymbol{S}_p \rangle = \frac{1}{2}\left(2 - \frac{3}{2}\right)\hbar^2 = \frac{1}{4}\hbar^2$

For S=0, $\langle \boldsymbol{S}_e \cdot \boldsymbol{S}_p \rangle = \frac{1}{2}\left(-\frac{3}{2}\right)\hbar^2 = -\frac{3}{4}\hbar^2$

So, the energy splitting $= \Delta E = a\left\{\frac{1}{4}\hbar^2 - \left(-\frac{3}{4}\hbar^2\right)\right\} = a\hbar^2$.

57. Ans: (b)

The first order correction to the energy is

$$\Delta E = \frac{\langle \psi_0 | H' | \psi_0 \rangle}{\langle \psi_0 | \psi_0 \rangle}; \quad \text{where } H' = bx^2 \ \text{ and } \psi_0 = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}.$$

It can be easily checked that ψ_0 is a normalized wave function.

$$\begin{aligned} \langle \psi_0 | \psi_0 \rangle &= \frac{1}{\pi a_0^3} \int_0^\infty e^{-2r/a_0} r^2 dr \int d\Omega = 1; \quad \int d\Omega = \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \\ &= \frac{4\pi}{\pi a_0^3} \int_0^\infty e^{-2r/a_0} r^2 dr = \frac{4}{a_0^3} \left(\frac{a_0}{2}\right)^3 \int_0^\infty e^{-x} x^2 dx = 1 \end{aligned}$$

Above, we have put $x = 2r/a_0$ and the integral $\int_0^\infty e^{-x} x^2 dx = 2!$.

Perturbation

$$H' = bx^2 = b(r \sin\theta \cos\phi)^2,$$

since $x = r\sin\theta\cos\phi$ in spherical polar coordinates.

We have

$$\begin{aligned} \int_0^{2\pi} \cos^2\phi d\phi &= \frac{1}{2}\int_0^{2\pi} (1 - \cos 2\phi) d\phi = \pi \\ \int_0^\pi \sin^2\theta \sin\theta d\theta &= -\int (1 - \cos^2\theta) d(cos\theta) = -\int_1^{-1} (1 - x^2) dx = \frac{4}{3} \end{aligned}$$

Therefore, perturbation energy is

$$\begin{aligned} \Delta E = \langle \psi_0 | bx^2 | \psi_0 \rangle &= \frac{b}{\pi a_0^3} \int e^{-2r/a_0} (r^2 \sin^2\theta \cos^2\phi) r^2 dr d\Omega \\ \int \sin^2\theta \cos^2\phi d\Omega &= \frac{4}{3}\pi. \end{aligned}$$

$$\begin{aligned}\Delta E &= \frac{b}{\pi a_0^3}\frac{4\pi}{3}\int_0^\infty e^{-2r/a_0}r^4 dr \\ &= \frac{4b}{3a_0^3}\left(\frac{a_0}{2}\right)^5\int_0^\infty e^{-x}x^4 dx \\ &= \frac{4b}{3a_0^3}\left(\frac{a_0}{2}\right)^5 (4!) = ba_0^2\end{aligned}$$

58. Ans: (d)
The following data are required:
Bohr magneton $\mu_B = 9.274 \times 10^{-24}$ JT^{-1}
Planck's constant $h = 6.626 \times 10^{-34}$ Js
Velocity of light $c = 3 \times 10^8$ m s^{-1}
The splitting of spectral lines in normal Zeeman effect is given by

$$\Delta E = \mu_B \mathcal{H} \implies h\Delta\nu = \mu_B\mathcal{H}$$
$$\nu = \frac{c}{\lambda} \implies \Delta\nu = -\frac{c}{\lambda^2}\Delta\lambda \implies \Delta\lambda = -\frac{\lambda^2}{c}\frac{\mu_B\mathcal{H}}{h}$$

Substituting the values $\lambda = 660$ nm $= 660 \times 10^{-9}$m and $\mathcal{H} = 0.3$ T, we get

$$\Delta\lambda = \frac{(660)^2 \times 9.274 \times 0.3}{3 \times 6.626} \times 10^{-16} \text{ m} = 6 \times 10^{-12} \text{ m} = 6 \text{ pm}$$

59. Ans: (c)
The normalized radial functions for hydrogen-like atom in the ground state ($1s$ state) is given by

$$R_{1s}(r) = 2\left(\frac{z}{a_0}\right)^{3/2} e^{-zr/a_0},$$

from which one can write down the initial radial wave function u_i of electron in $1s$ state of ^3H atom and the final radial wave function v_f of electron in $1s$ state of ^3He atom.

$$\begin{aligned}u_i &= 2\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}; \\ v_f &= 2\left(\frac{2}{a_0}\right)^{3/2} e^{-2r/a_0};\end{aligned}$$

Applying the sudden approximation to the sudden change in the atomic state of the electron arising from beta decay, we find the transition probability.

$$\langle v_f|u_i\rangle = \frac{8\sqrt{2}}{a_0^3}\int_0^\infty e^{-3r/a_0}r^2 dr.$$

The integral can be carried out easily.

$$\int_0^\infty e^{-\alpha r} r^2 dr = \frac{2}{\alpha^3}.$$

The result is

$$\begin{aligned} \langle v_f | u_i \rangle &= \frac{8\sqrt{2}}{a_0^3}\frac{2}{\alpha^3}, \qquad \text{with } \alpha = \frac{3}{a_0} \\ &= \frac{16\sqrt{2}}{27} = 0.838. \end{aligned}$$

The probability of finding the final state in the ground state of ^3He atom is

$$P = |\langle v_f | u_i \rangle|^2 = 0.838^2 = 0.702.$$

60. Ans: (a)
The commutator $[\boldsymbol{\sigma}\cdot\boldsymbol{p}, H]_-$ can be evaluated as shown below:.

$$\begin{aligned} [\boldsymbol{\sigma}\cdot\boldsymbol{p}, H]_- &= [\boldsymbol{\sigma}\cdot\boldsymbol{p}, c(\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta\mu)]_- \\ &= c[\boldsymbol{\sigma}\cdot\boldsymbol{p}, \boldsymbol{\alpha}\cdot\boldsymbol{p}]_- \\ &= c\left\{ \begin{bmatrix} \boldsymbol{\sigma}\cdot\boldsymbol{p} & 0 \\ 0 & \boldsymbol{\sigma}\cdot\boldsymbol{p} \end{bmatrix} \begin{bmatrix} 0 & \boldsymbol{\sigma}\cdot\boldsymbol{p} \\ \boldsymbol{\sigma}\cdot\boldsymbol{p} & 0 \end{bmatrix} \right. \\ &\qquad \left. - \begin{bmatrix} 0 & \boldsymbol{\sigma}\cdot\boldsymbol{p} \\ \boldsymbol{\sigma}\cdot\boldsymbol{p} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\sigma}\cdot\boldsymbol{p} & 0 \\ 0 & \boldsymbol{\sigma}\cdot\boldsymbol{p} \end{bmatrix} \right\} \\ &= c\left\{ \begin{bmatrix} 0 & (\boldsymbol{\sigma}\cdot\boldsymbol{p})^2 \\ (\boldsymbol{\sigma}\cdot\boldsymbol{p})^2 & 0 \end{bmatrix} - \begin{bmatrix} 0 & (\boldsymbol{\sigma}\cdot\boldsymbol{p})^2 \\ (\boldsymbol{\sigma}\cdot\boldsymbol{p})^2 & 0 \end{bmatrix} \right\} = 0. \end{aligned}$$

61. Ans: (c)
Let ψ_1 and ψ_2 denote positive energy states and ψ_3 and ψ_4 denote negative energy states. Then

$$\begin{aligned} (c\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2)\psi_{1,2} = |E|\psi_{1,2}; &\qquad (c\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2)\psi_{3,4} = -|E|\psi_{3,4} \\ \Lambda_+\psi_{1,2} = 1; \quad \Lambda_+\psi_{3,4} = 0; &\qquad \Lambda_-\psi_{1,2} = 0; \quad \Lambda_-\psi_{3,4} = 1. \end{aligned}$$

Please also refer to Solved Problem (11.2) & (11.3); Pages 311 & 312 of Q.M. book by V. Devanathan .
The correct answer is: (c).

62. Ans: (a)
Conductivity $\sigma = e(n_e\mu_e + n_h\mu_h)$
In an intrinsic semiconductor, $n_e = n_h = n_i$.

So, $\sigma = n_i e(\mu_e + \mu_h)$.
Substituting the values, $n_i = 3.6 \times 10^{19}$, $\mu_e = 0.54$. $\mu_h = 0.18$ and $e = 1.6 \times 10^{-19}$, we get

$$\sigma = 3.6 \times 10^{19} \times 1.6 \times 10^{-19}(0.54 + 0.18) = 4.1472\ \Omega^{-1}\text{m}^{-1}.$$

63. Ans: (c)
According to the Nuclear Shell Model, the nucleon configuration is:

$$^{27}\text{Al}_{13} : \left(1s_{1/2}\right)^{2p,2n} \left(1p_{3/2}\right)^{4p,4n} \left(1p_{1/2}\right)^{2p,2n} \left(1d_{5/2}\right)^{5p,6n}$$

According to the Schmidt model, only the odd nucleon contributes to the magnetic moment.

$$\text{Magnetic moment} = j\left\{g_l \pm (g_s - g_l)\frac{1}{2l+1}\right\},\ j = l \pm \frac{1}{2}.$$

For the odd proton in the state $1d_{5/2}$, $j = 5/2, l = 2$.

$$\text{Magnetic Moment} = \frac{5}{2}\left\{1 + (5.586 - 1)\frac{1}{5}\right\} = 4.793\,\mu_n.$$

64. Ans:(b)
At equilibrium,
Number of transitions from $|0\rangle$ to $|1\rangle$ = Number of transitions from $1\rangle$ to $|0\rangle$.
$P_0 n W_{0\to 1} = P_1 W_{1\to 0} + P_1 n W_{1\to 0}$.
Induced absorption= spontaneous emission + Induced emission, since there cannot be any spontaneous absorption.

65. Ans: (d)
For (A,B) = (1,0), $Y' = 1,\ Y'' = 0,\ Y''' = 1,\ Y = 1$.
For (A,B) = (1,1), $Y' = 0,\ Y'' = 1,\ Y''' = 1,\ Y = 0$.
For (A,B) = (0,0), $Y' = 1,\ Y'' = 1,\ Y''' = 1,\ Y = 0$.