THE ACADEMY OF SCIENCES, CHENNAI

Physics Olympiad for P.G. Students - 2019

Instructions to the candidates. Please read carefully.

1. The question paper is divided into two parts - PART A and PART B - and carries **a total of 150 marks.** PART A carries 50 marks and PART B carries 100 marks.

2. PART A consists of 30 questions, out of which only 25 are to be answered. Each question carries **two marks**. A wrong answer carries a **negative mark of** $-\frac{1}{2}$.

3. PART B consists of 35 questions, out of which only 25 are to be answered. Each question carries **four marks**. A wrong answer carries **a negative mark of** -1.

4. Each question carries four responses (answers). Choose the correct response **('A' or 'B' or 'C' or 'D')** and enter the same in ink in the BOX provided in the ANSWER SHEET.

5. **Never overwrite** in the Box but strike out and rewrite, if you wish to make corrections. Put a short signature (initials) to authenticate your correction.

6. **Only a simple non-programmable calculator is allowed but exchange of calculators among the candidates is not permitted. Use of any other electronic gadgets (including mobile phone) is not permitted.**

7. Please find attached numerical values of some Fundamental Physical Constants.

8. For rough work, use only the blank sheets attached at the end of the question paper.

9. **In the given Answer Sheet, please enter your Name, Registration Number, Course of Study, Institution, Email/Mobile No. along with your signature.**

Some Fundamental Physical Constants

Physical Quantity	Symbol	Value
Speed of light	c	2.998×10^{8} m s^{-1}
Elementary charge	e	1.602×10^{-19} C 4.803×10^{-10} e.s.u.
Planck constant	h	6.626×10^{-34} Js
Planck constant/2π	$\hbar$	1.055×10^{-34} Js
Electron rest mass	m_e	9.109×10^{-31} kg 0.511 MeV/c^2
Proton rest mass	m_p	1.673×10^{-27} kg 938.3 MeV/c^2
Neutron rest mass	m_n	1.675×10^{-27} kg 939.6 MeV/c^2
Avagadro's number	N_A	6.0221×10^{23} mol^{-1} 6.0221×10^{26}(Kg.mol)$^{-1}$
Universal gas constant	$R = N_A k_B$	8.3145 J/mol.K
Fine structure constant α	$1/\alpha$	137.036
Bohr magneton	μ_B	9.274×10^{-24} JT^{-1}
Nuclear magneton	μ_n	5.051×10^{-27} JT^{-1}
Permittivity of vacuum	ϵ_0	8.854×10^{-12} F m^{-1}
Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Boltzmann constant	k_B	1.381×10^{-23} J K^{o-1}
Electron Volt	1 eV	1.602×10^{-19} J

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Time: 3 hours **Max. Marks: 150**

PART A

Answer any 25 out of 30 questions. Each question carries 2 marks.
A wrong answer carries a negative mark of $-\frac{1}{2}$.

1. If $r = |\boldsymbol{r}|$, then $\boldsymbol{\nabla} r^3$ is equal to

(a) $3r^2$, (b) $3r^2\boldsymbol{r}$, (c) $3r\boldsymbol{r}$, (d) $3\boldsymbol{r}$.

2. If the vectors $\boldsymbol{a} = \boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{b} = 2\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{c} = \lambda\boldsymbol{i} + \boldsymbol{j} + \mu\boldsymbol{k}$ are mutually orthogonal, then $(\lambda, \mu) =$

(a) $(-2, 3)$, (b) $(3, -2)$, (c) $(-3, 2)$, (d) $(2, -3)$.

3. Given 15 players, you are required to choose a team of 11 players. The number of different teams that can be formed is:

(a) $\frac{15!}{4!}$ (b) $\frac{15!}{4!}$ (c) $\frac{15!}{11!\,4!}$ (d) $\frac{15!\,4!}{11!}$

4. According to the special theory of relativity, the speed v of a free particle of rest mass m_0 and total energy E is:

(a) $v = c\left(1 - \frac{m_0c^2}{E}\right)^{1/2}$ (b) $v = \sqrt{\frac{2E}{m_0}}\left(1 + \frac{m_0c^2}{E}\right)$

(c) $v = c\left(1 - \left(\frac{m_0c^2}{E}\right)^2\right)^{1/2}$ (d) $v = c\left(1 + \frac{m_0c^2}{E}\right)$

5. The eigenvalues of the matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ are

(a) 0, 0 (b) 0, 1 (c) 1, 1 (d) 1, -1.

6. Given the position vector $\boldsymbol{r}$ and the Pauli spin vector $\boldsymbol{\sigma}$, which of the following is a pseudoscalar?

(a) $\boldsymbol{r}\cdot\boldsymbol{r}$, (b) $\boldsymbol{\sigma}\cdot\boldsymbol{r}$, (c) $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$, (d) $(\boldsymbol{\sigma}_1\cdot\boldsymbol{r})(\boldsymbol{\sigma}_2\cdot\boldsymbol{r})$

7. Using the residue theorem, evaluate the integral

$$\oint_c \frac{3z^2 + z}{z^2 - 1} dz,$$

where c is the circle with $|z| = 2$.

(a) πi (b) $2\pi i$ (c) $3\pi i$ (d) $4\pi i$

8. Consider two particles with rest mass m_1 and m_2 moving with the same kinetic energy E. If λ_1 and λ_2 are their de Broglie wavelengths, then the ratio λ_1/λ_2 will be

(a) 1 (b) $\frac{m_1}{m_2}$ (c) $\frac{m_2}{m_1}$ (d) $\sqrt{\frac{m_2}{m_1}}$

8. Ans: (d)
de Broglie wavelength $\lambda = \frac{h}{p}$.
Since the kinetic energy $E = \frac{p^2}{2m}$, $p = (2mE)^{1/2}$. Therefore $\frac{\lambda_1}{\lambda_2} = \left(\frac{m_2}{m_1}\right)^{1/2}$.

9. Given the x-component of momentum, p_x and y-component of orbital angular momentum, L_y; the commutator $[p_x^2, L_y]_-$ is

(a) 0, (b) $2i\hbar p_x p_z$ (c)$-2i\hbar p_x p_y$, (d) $2i\hbar p_y p_z$.

10. There are two uniformly charged spheres, each of radius 5 cm and with charge 6.5×10^{-7} Coulomb; their centres being separated by a distance 0.5 metre. Given that $1/(4\pi\epsilon_0) = 9 \times 10^9$ N m^2 C^{-2}, where ϵ_0 denotes the permittivity of vacuum, the electrostatic force between them is:

(a) 15.21×10^{-3} N (b) 7.61×10^{-3} N (c) 23.77×10^{-3} N (d) 9.51×10^{-3}N

11. The pressure P of a fluid is related to its number density ρ by the equation of state $P = a\rho + b\rho^2$, where a and b are constants. If the initial volume of the fluid is V_0, the work done on the system when it is compressed isothermally so as to increase the number density from an initial value ρ_0 to $2\rho_0$ is

(a) $a\rho_0 V_0$ (b) $(a + b\rho_0)\rho_0 V_0$ (c) $\left(\frac{3a}{2} + \frac{7\rho_0 b}{3}\right)\rho_0 V_0$ (d) $(a \ln 2 + b\rho_0)\rho_0 V_0$

12. A particle moves in an elliptical orbit $x^2 + 4y^2 = 8$. At a particular time, it is at the point $x = 2, y = 1$ and its x-component of velocity is 6 units. Then its y-component of velocity in the same units should be

(a) -3, (b) -2, (c) 1, (d) 4.

13. A particle of mass m moves in one dimension in the potential $V = \frac{1}{2}k(t)x^2$, where $k(t)$ is a time dependent parameter. Then the rate of change $\frac{d}{dt}\langle V\rangle$ of the expectation value $\langle V\rangle$ of the potential energy is

(a) $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle + \frac{k}{2m}\langle xp + px\rangle$, (b) $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle + \frac{1}{2m}\langle p^2\rangle$,
(c) $\frac{k}{2m}\langle xp + px\rangle$, (d) $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle$.

14. In a strong interaction, π^- in s-state is captured at rest by deuteron resulting in two neutrons in the final state.

$$\pi^- + d \longrightarrow n + n$$

If the parity of π^-, d and n are -1, $+1$, $+1$ respectively, then the spin S of the final state of two neutrons is
(a) $S = 0$ (b) $S = 1$ (c) either $S = 0$ or $S = 1$
(d) reaction not allowed in strong interaction

15. If the radius of the nucleus $^{64}\mathrm{Zn}_{30}$ is 4.8×10^{-13} cm, then the radius of nucleus $^{27}\mathrm{Al}_{13}$ is expected to be
(a) 2.8×10^{-13} cm, (b) 3.6×10^{-13} cm,
(c) 5.2×10^{-13} cm, (d) 8.6×10^{-13} cm

16. According to the Nuclear Shell Model, the spin-parity (J^π) of the ground state and the first excited state of the nucleus $^{17}O_8$ are respectively
(a) $\frac{5}{2}^+$, $\frac{3}{2}^+$ (b) $\frac{3}{2}^+$, $\frac{5}{2}^+$ (c) $\frac{5}{2}^+$, $\frac{1}{2}^+$ (d) $\frac{5}{2}^-$, $\frac{1}{2}^-$

17. The nucleus ^{164}Er exhibits rotational energy spectrum. If the first excited state is of energy 90 KeV, the energy of the second excited state will be approximately
(a) 300 KeV (b) 270 KeV (c) 180 KeV (d) 135 KeV

18. The Hamiltonian corresponding to the Lagrangian

$$L = a\dot{x}^2 + b\dot{y}^2 - kxy$$

is
(a) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} + kxy$ (b) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} - kxy$
(c) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} + kxy$ (d) $\frac{p_x^2+p_y^2}{4ab} + kxy$

19. Two light pulses are emitted simultaneously at A and B, separated by a distance of 10 km. For a person traveling on a magic carpet which is moving in the direction of AB with a relativistic velocity 0.95 c, they will not appear to be simultaneous? Which pulse emitted at A or B will appear to be emitted earlier? What will be the time difference?
(a) A, 1.014×10^{-4} sec (b) B, 1.014×10^{-4} sec
(c) A, 1.014×10^{-7} sec (d) B, 1.014×10^{-7} sec

20. Given the following data:
Density of NaCl = 2170 Kg/m^3,
Molecular weight of NaCl = 58.45,
Avagadro number $N_A = 6 \times 10^{26}$ per Kg. mole,
the latice constant of Nacl crystal is found to be
(a) 1.41 Å (b) 2.82 Å (c) 4.23 Å (d) 5.64 Å

21. According to Debye's theory, the specific heat of solids at low temperatures depends on the absolute temperature T in such a way that it is proportional to
(a) T (b) T^2 (c) T^3 (d) $e^{-1/T}$

22. Germanium is an intrinsic semiconductor. The impurity that has to be added to make it a p-type semiconductor is
(a) Aluminium (b) Antimony (c) Arsenic (d) Phosphorous

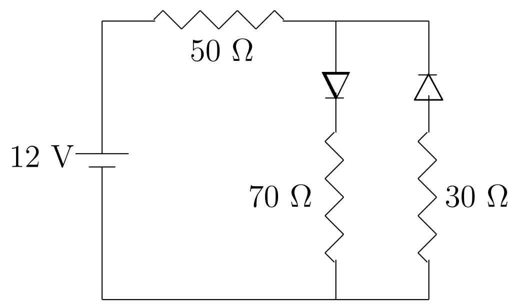
23. In a n-type semiconductor, the position of the Fermi level
(a) is lower than the centre of energy gap
(b) is at the centre of the energy gap
(c) is higher than the centre of energy gap
(d) can be anywhere depending upon the electron concentration.

24. For a given uniform magnetic field $\boldsymbol{B} = B_0 \hat{\boldsymbol{k}}$, where B_0 is a constant, a possible choice for the electromagnetic vector potential $\boldsymbol{A}$ is
(a) $B_0\, y\hat{\boldsymbol{i}}$ (b) $-B_0\, y\hat{\boldsymbol{i}}$ (c) $B_0(x\hat{\boldsymbol{j}} + y\hat{\boldsymbol{i}})$ (d) $B_0(x\hat{\boldsymbol{i}} - y\hat{\boldsymbol{j}})$

25. In the scattering of some elementary particles, the scattering cross section σ is found to depend on the total energy E and the fundamental constants h (Planck's constant) and c (the speed of light in vacuum). Using the dimensional analysis, the dependence of σ on these quantities is given by

(a) $\sqrt{\frac{\mathrm{hc}}{\mathrm{E}}}$ (b) $\frac{\mathrm{hc}}{\mathrm{E}^{1/2}}$ (c) $\left(\frac{\mathrm{hc}}{\mathrm{E}}\right)^2$ (d) $\frac{\mathrm{hc}}{\mathrm{E}}$

26. In the circuit diagram, given below, involving two ideal diodes,



the voltage drop across the resistance 50 Ω is

(a) 2.38 Volts (b) 5 Volts (c) 7.5 Volts (d) 12 Volts

27. In Boolean algebra $\overline{\left(A+\bar{B}\right)\cdot\bar{C}}$ will be equal to

(a) $(\bar{A}\cdot B)+\bar{C}$ (b) $(A\cdot\bar{B})+C$ (c) $(A\cdot B)\cdot C$ (d) $(A+\bar{B})+C$

28. The orbital angular angular momentum L of two protons in the triplet spin state is restricted to

(a) $L=0$ (b) $L=1$ (c) L odd (d) L even

29. Ω^- is a missing member of the 10-fold baryon group with spin-parity $J^\pi=\frac{3}{2}^-$ that was discovered by observing its decay modes. Which one of the following decay modes is not possible?

(a) Λ^0+K^- (b) $\Lambda^0+\pi^-$ (c) $\Xi^0+\pi^-$ (d) $\Xi^-+\pi^0$

30. The elementary particle Ξ^- is made up of three quarks as given below:

(a) dss (b) $\bar{u}\bar{u}\bar{s}$ (c) dss (d) sss

PART B

Answer any 25 out of 35 questions. Each question carries 4 marks. A wrong answer carries a negative mark of −1.

31. If $y=\frac{1}{\tanh(x)}$, then x is

(a) $\ln\left(\frac{y+1}{y-1}\right)$ (b) $\ln\left(\frac{y-1}{y+1}\right)$ (c) $\ln\sqrt{\frac{y-1}{y+1}}$ (d) $\ln\sqrt{\frac{y+1}{y-1}}$

32. A person measures the period of oscillations of a simple pendulum inside a stationary lift and finds it to be T. If the lift starts accelerating upwards with an acceleration $g/4$, then the time period of oscillation will be:

(a) $2T/\sqrt{5}$ (b) $\sqrt{5}T/2$ (c) $\sqrt{5}/(2T)$ (d) $2/(\sqrt{5}\,T)$

33. Using the standard integral $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$, the Fourier transform

$$F(\boldsymbol{k}) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} f(\boldsymbol{r}) e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} d^3 r$$

of a function $f(\boldsymbol{r}) = e^{-r^2/a^2}$ where $r^2 = x^2 + y^2 + z^2$ is obtained as

(a) $\left(\frac{a}{\sqrt{2}}\right)^3 e^{-k^2a^2/4}$ (b) $\frac{1}{(2\pi)^{3/2}} e^{-k^2a^2/4}$ (c) $\left(\frac{a}{\sqrt{2\pi}}\right)^{3/2} e^{-k^2a^2/2}$ (d) $\left(\frac{a}{\sqrt{2\pi}}\right)^{3/2} e^{-k^2a^2}$

34. Solve the following differential equation:

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0.$$

Find the value of x at $t = 2$, given that $x = 0$ at $t = 0$ and $x = 1$ at $t = 1$.

(a) $e^2 + 1$ (b) $e^2 + e$ (c) $e + 2$ (d) $2e$

35. Consider the matrix equation

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & b & 2c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The condition for existence of a non-trivial solution and the corresponding normalized solution are:

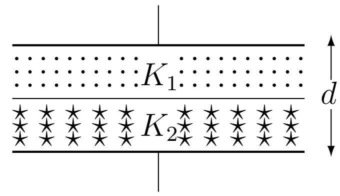
(a) b = 2c and $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ (b) c = 2b and $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

(c) c = b + 1 and $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{\sqrt{6}} \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$ (d) b = c + 1 and $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

36. A monoatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionl;ess piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion respectively, then T_1/T_2 is given by

(a) $\left(\frac{L_1}{L_2}\right)^{2/3}$ (b) $\left(\frac{L_2}{L_1}\right)^{2/3}$ (c) $\frac{L_1}{L_2}$ (d) $\frac{L_2}{L_1}$

37. A parallel plate capacitor of plate area A, separation d is filled with two dielectrics of equal thickness as shown in the given figure. If the dielectric constants are K_1 and K_2, the net capacitance of the capacitor is



(a) $\frac{\epsilon_0 \mathrm{A}}{\mathrm{d}}(\mathrm{K}_1 + \mathrm{K}_2)$ (b) $\frac{\epsilon_0 \mathrm{A}}{\mathrm{d}}\left(\frac{\mathrm{K}_1 + \mathrm{K}_2}{\mathrm{K}_1 \mathrm{K}_2}\right)$

(c) $\frac{2\epsilon_0 \mathrm{A}}{\mathrm{d}}\left(\frac{\mathrm{K}_1 \mathrm{K}_2}{\mathrm{K}_1 + \mathrm{K}_2}\right)$ (d) $\frac{2\epsilon_0 \mathrm{A}}{\mathrm{d}}\left(\frac{\mathrm{K}_1 + \mathrm{K}_2}{\mathrm{K}_1 \mathrm{K}_2}\right)$

38. A proton of kinetic energy 1 MeV is moving in a circular path of radius R in uniform magnetic field. What should be the kinetic energy of α - particle to describe a circle of the same radius in the same field?

(a) 4 MeV (b) 2 MeV (c) 1 MeV 0.5 MeV

39. The speed of the electromagnetic wave travelling in a medium of relative permeability 1.3 and relative permittivity 2.14 is

(a) 3.6×10^6 m s^{-1} (b) 1.8×10^6 m s^{-1}
(c) 3.6×10^8 m s^{-1} (d) 1.8×10^8 m s^{-1}

40. Two slits 4 mm apart are illuminated by light of wavelength 6000 Å. What will be the fringe width on a screen placed 2 m from the slits?

(a) 0.12 mm (b) 0.3 mm (c) 3.0 mm (d) 4.0 mm

41. The mean lives of a radioactive substance are 1620 years and 405 years for α-emission and β-emission respectively. The time during which three fourth of a sample will decay if it is decaying both by α-emisiion and β-emission simultaneously is:

(a) 339 years (b) 449 years (c) 559 years (d) 669 years

42. A particle of mass m kept in a potential $V(x) = -\frac{1}{2}kx^2 + \frac{1}{4}\lambda x^4$ (where k and λ are positive constants) undergoes small oscillations about an equilibrium point. The frequency of small oscillations is

(a) $\frac{1}{2\pi}\sqrt{\frac{2\lambda}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{\lambda}{m}}$

43. If $\boldsymbol{J}$ ia an angular momentum vector operator and $\boldsymbol{A}$ is any polar vecor, then the matrix element $\langle j, m+1|\boldsymbol{J}\cdot\boldsymbol{A}|j, m\rangle$ is equal to

(a) $\frac{1}{2}(A_x - iA_y)\{(j-m)(j+m+1)\}^{1/2}\,\hbar$

(b) $\frac{1}{2}(A_x + iA_y)\{(j-m)(j+m+1)\}^{1/2}\,\hbar$

(c) $\frac{1}{2}(A_x - iA_y)\{(j+m)(j-m+1)\}^{1/2}\,\hbar$

(d) $\frac{1}{2}(A_x + iA_y)\{(j+m)(j-m+1)\}^{1/2}\,\hbar$.

44. The state of a particle of mass m in a one-dimensional rigid box in the interval 0 to L is given by the normailized wave function

$$\psi(x) = \sqrt{\frac{2}{L}}\left(\frac{3}{5}\sin\frac{2\pi x}{L} + \frac{4}{5}\sin\frac{4\pi x}{l}\right).$$

If its energy is measured, the possible outcomes and the average value of energy are respectively

(a) $\frac{h^2}{2mL^2}, \frac{2h^2}{mL^2}$ and $\frac{73}{50}\frac{h^2}{mL^2}$ (b) $\frac{h^2}{8mL^2}, \frac{h^2}{2mL^2}$ and $\frac{19}{40}\frac{h^2}{mL^2}$

(c) $\frac{h^2}{2mL^2}, \frac{2h^2}{mL^2}$ and $\frac{19}{10}\frac{h^2}{mL^2}$ (d) $\frac{h^2}{8mL^2}, \frac{2h^2}{mL^2}$ and $\frac{73}{200}\frac{h^2}{mL^2}$

45. There are three long parallel conductors A, B and C, each of length 10 metres but with a distance of separation of 10 cm. If the currents that flow through the conductors A,B,C are 2 amp, 3 amp and 4 amp respectively and they are in the same direction, the net force on the middle conductor B (given that $\mu_0/(4\pi) = 10^{-7}$ henry/meter, where μ_0 denotes the permeability of vacuum) is

(a) 1.2×10^{-4} N (b) 2.4×10^{-4} N (c) 3.6×10^{-4} N (d) 4.8×10^{-4} N

46. A solid sphere of radius R has a charge density

$$\rho(r) = \rho_0\left(1 - \frac{ar}{R}\right),$$

where r is the radial coordinate and ρ_0, a and R are positive constants. If the magnitude of the electric field at $r = R/2$ is 1.25 times that at $r = R$, then the value of a is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

47. A solenoid of 2 metre length and of diameter 3 cm has 1000 turns of winding. If it carries a current of 5 Amps, the magnetic induction B in units (Tesla) at its centre along its axis is

(a) 3.14×10^{-3} (b) 6.28×10^{-3} (c) 6.28×10^{-5} (d) 3.14×10^{-5}

48. Find the threshold energy (in MeV) of $\bar{p}$ in the laboratory frame for the reaction

$$\bar{p} + p \longrightarrow \Lambda + \bar{\Lambda}.$$

It is given that the mass of the proton(antiproton) is 938 MeV/c^2 and the mass of Λ ($\bar{\Lambda}$) is 1115 MeV/c^2.

(a) 354 (b) 555 (c) 775 (d) 925

49. In a betatron with a maximum magnetic field B and orbital radius R, the maximum energy E of electron of charge e (assuming that its velocity is nearly equal to the velocity of light c) that it can deliver is:

(a) $E = RBec$ (b) $E = Bec$ (c) $E = Bec/R$ (d) $E = BeR/c$

50. The Fermi energy of a monovalent bcc solid whose lattice constant a is 5.34 $\AA$ is

(a) 1.02 eV (b) 2.03 eV (c) 3.05 eV (d) 4.06 eV

51. If ϵ_F is the Fermi energy of an electron gas confined within a volume V, then the average energy of an electron at absolute zero temperature is:

(a) $\frac{1}{2}\epsilon_F$ (b) $\frac{2}{3}\epsilon_F$ (c) $\frac{2}{5}\epsilon_F$ (d) $\frac{3}{5}\epsilon_F$

52. The radial wave function of electron in the $2p$ state of hydrogen atom is given by

$$R_{21}(r) = \frac{1}{2\sqrt{6}}\left(\frac{1}{a}\right)^{3/2} e^{-r/(2a)}(r/a)$$

where a is a constant. If the radial probability distribution of electron is maximum at the radial distance $r = r_0$, then

(a) $r_0 = a$, (b) $r_0 = \frac{3}{2}a$ (c) $r_0 = 3a$ (d) $r_0 = 4a$

53. Consider a particle of mass m in an attractive delta function potential $\alpha\delta(x)$ with the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x).$$

Given the definite integrals

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{a}}; \qquad \int_{-\infty}^{\infty} x^2 e^{-\alpha x^2} dx = \frac{1}{2\alpha}\sqrt{\frac{\pi}{\alpha}};$$

find the ground state energy of the particle using the variational principle with the trial wave function $\psi(x) = A\exp(-bx^2)$.

(a) $-\frac{m\alpha^2}{\pi\hbar^2}$ (b) $-\frac{2m\alpha^2}{\pi\hbar^2}$ (c) $-\frac{m\alpha^2}{2\hbar^2}$ (d) $\frac{m\alpha^2}{\pi\hbar^2}$

54. The normalized wave functions $|0\rangle$ and $|1\rangle$ of the ground state and the first excited state of the one-dimensional harmonic oscillator are

$$|0\rangle = \left(\frac{\alpha}{\sqrt{\pi}}\right)^{1/2} e^{-(1/2)\alpha^2 x^2}; \quad |1\rangle = \left(\frac{2\alpha^3}{\sqrt{\pi}}\right)^{1/2} x\, e^{-(1/2)\alpha^2 x^2},$$

where $\alpha^2 = m\omega/\hbar$, m denoting the mass of the oscillating particle with angular frequency ω. If the particle is in a mixture of states

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle,$$

where c_0 and c_1 are constants such that $c_0^2 + c_1^2 = 1$, find the value of c_0 for which expectation value $\langle x\rangle$ is maximum and also the maximum value of $\langle x\rangle$.

(a) $c_0 = \frac{1}{\sqrt{2}}$, $\langle x\rangle = \left(\frac{\hbar}{m\omega}\right)^{1/2}$ (b) $c_0 = \frac{1}{2}$, $\langle x\rangle = \left(\frac{\hbar}{2m\omega}\right)^{1/2}$

(c) $c_0 = \frac{1}{\sqrt{2}}$, $\langle x\rangle = \left(\frac{\hbar}{2m\omega}\right)^{1/2}$ (d) $c_0 = \frac{1}{2}$, $\langle x\rangle = \left(\frac{\hbar}{m\omega}\right)^{1/2}$

55. Consider the decay of K^- meson in its rest frame.

$$K^- \longrightarrow \mu^- + \bar{\nu}_\mu.$$

If the rest masses of K^- meson and μ^- are 494 MeV/c^2 and 106 MeV/c^2 respectively, then the energy of the out-going $\bar{\nu}_\mu$ (assuming it to be massless) is approximately

(a) 120 MeV (b) 236 MeV (c) 300 MeV (d) 388 MeV

56. The effective spin-spin interaction between the electron spin $\boldsymbol{S}_e$ and the proton spin $\boldsymbol{S}_p$ in the ground state of the hydrogen atom is given by $H' = a\,\boldsymbol{S}_e \cdot \boldsymbol{S}_p$. As a result of this interaction, the energy levels split by an amount

(a) $\frac{1}{2}ah^2$ (b) $2ah^2$ (c) ah^2 (d) $\frac{3}{2}ah^2$

57. The Coulomb potential $V(r) = -e^2/r$ of a hydrogen atom is perturbed by adding $H' = bx^2$ (where b is a constant) to the Hamiltonian. The first order correction to the ground state energy is (The ground state wave function is $\psi_0 = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$.)

(a) $2ba_0^2$ (b) ba_0^2 (c) $ba_0^2/2$ (d) $\sqrt{2}ba_0^2$

58. In a normal Zeeman effect experiment, using a magnetic field of 0.3 T, the splitting between the components of a 660 nm spectral line is

(a) 12 pm (b) 10 pm (c) 8 pm (d) 6 pm

59. The tritium atom ^3H in the ground state decays into Helium atom ^3He by beta decay.

$$^3H \to {}^3He + e^- + \bar{\nu}_e.$$

Given the normalized radial function $R_{nl}(r)$ for hydrogen-like atom of charge z in the ground state ($1s$ state) and the standard integral I

$$R_{1s}(r) = 2\left(\frac{z}{a_0}\right)^{3/2} e^{-zr/a_0}, \qquad I = \int_0^\infty e^{-\alpha r} r^n dr = \frac{n!}{\alpha^{n+1}}, (n \text{ positive integer})$$

the probability that the Helium atom will be in the ground state after the transition, using the sudden approximation, is:

(a) 0.52, (b) 0.60 (c) 0.70 (d) 0.84

60. If $\boldsymbol{\sigma}$ is the Pauli spin vector, $\boldsymbol{p}$ is the momentum vector and $H = c\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2$ is the Dirac Hamiltonian, then the commutator $[\boldsymbol{\sigma}\cdot\boldsymbol{p}, H]_-$ is

(a) 0 (b) $-i\hbar c(\boldsymbol{\alpha}\times\boldsymbol{p})$ (c) $c(\boldsymbol{\sigma}\cdot\boldsymbol{p})(\boldsymbol{\alpha}\cdot\boldsymbol{p})$ (d) $c(\boldsymbol{\alpha}\cdot\boldsymbol{p})(\boldsymbol{\sigma}\cdot\boldsymbol{p})$

61. If Λ_+ and Λ_- are the projection operators for the positive and negative energy states of the Dirac Hamiltonian $H = c\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2$, then they are given by

$$(a) \quad \Lambda_+ = 1 + \frac{c\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2}{|E|}, \qquad \Lambda_- = 1 - \frac{c\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2}{|E|}$$

$$(b) \quad \Lambda_+ = 1 - \frac{c\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2}{|E|}, \qquad \Lambda_- = 1 + \frac{c\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2}{|E|}$$

$$(c) \quad \Lambda_+ = \frac{1}{2}\left(1 + \frac{c\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2}{|E|}\right), \qquad \Lambda_- = \frac{1}{2}\left(1 - \frac{c\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2}{|E|}\right)$$

$$(d) \quad \Lambda_+ = \sqrt{\frac{1}{2}}\left(1 + \frac{c\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2}{|E|}\right), \qquad \Lambda_- = \sqrt{\frac{1}{2}}\left(1 - \frac{c\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2}{|E|}\right)$$

62. In a sample of intrinsic germanium at room temperature, the mobilities of electrons and holes are 0.54 $\mathrm{m^2V^{-1}s^{-1}}$ and 0.18 $\mathrm{m^2V^{-1}s^{-1}}$ respectively. If the electron and hole densities are equal to $3.6 \times 10^{19}\,\mathrm{m^{-3}}$, the germanium conductivity in MKS units is

(a) 4.14 (b) 2.12 (c) 1.13 (d) 5.6

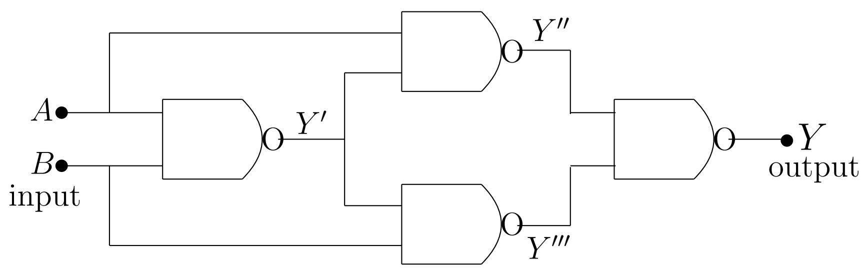
63. According to the nuclear shell model, the magnetic moment of the $^{27}\mathrm{Al}_{13}$ nucleus in nuclear magneton (μ_n) is (Given that for a proton $g_l = 1\,\mu_n$, $g_s = 5.586\,\mu_n$ and for a neutron $g_l = 0$, $g_s = -3.826\,\mu_n$.)

(a) $-1.913\,\mu_n$ (b) $14.414\,\mu_n$ (c) $4.793\,\mu_n$ (d) 0

64. Consider a system of identical atoms in equilibrium with black body radiation in a cavity at temperature T. The equilibrium probabilities for each atom being in the ground state $|0\rangle$ and an excited state $|1\rangle$ are P_0 and P_1 respectively. Let n be the average number of photons in a mode in the cavity that causes transition between the two states. Let $W_{0\to1}$ and $W_{1\to0}$ denote respectively the square of the matrix elements corresponding to the atomic transitions $|0\rangle \to |1\rangle$ and $|1\rangle \to |0\rangle$. Which of the following equations hold in equilibrium?

(a) $P_0 n W_{0\to1} = P_1 W_{1\to0}$ (b) $P_0 n W_{0\to1} = P_1 W_{1\to0} + P_1 n W_{1\to0}$
(c) $P_0 W_{0\to1} = P_1 n W_{1\to0}$ (d) $P_0 n W_{0\to1} = P_1 W_{1\to0} - P_1 n W_{1\to0}$

65. For the three pairs of given inputs (A,B) = (1,0), (1,1), (0,0), the output Y of the combination of gates as shown below are respectively



(a) 0, 1, 1 (b) 1, 0, 1 (c) 1, 1, 1 (d) 1, 0, 0