How do the particles acquire mass?

The Gauge Theories, Higgs Field and Higgs Bosons

V. Devanathan*
The Academy of Sciences, Chennai
Department of Nuclear Physics, University of Madras
Guindy Campus, Chennai - 600 025

Received 20 August 2019

Abstract: The particles interact by exchange of vector bosons known as gauge bosons. The electromagnetic interaction is due to exchange of photons which are massless but the weak interaction is due to exchange of intermediate vector bosons which are massive. It is postulated that the vector bosons acquire mass by interaction with the hypothetical scalar energy field known as Higgs field that pervades the entire universe. This concept has been extended to the generation of mass for leptons and quarks which are the constituents of matter.

1 Introduction

How do the particles acquire mass? You will find a satisfactory answer in Quantum Field Theory [1,2] which is based on the Lagrangian formulation of classical mechanics and the Hamiltonian action principle.

Consider the Lagrangian of a system of particles, which is a function of their positions and velocities that are time-dependent. The insistence that it should obey the Hamilton principle of least action leads to the Euler-Lagrange equation of motion. This Lagrangian formalism developed for a discrete mechanical system can be extended to fields which are continuous functions of space-time coordinates and their first derivatives. Using the action integral and the variational principle, we can obtain analogous Euler-Lagrangian equations of motion for the fields.

The Schrödinger equation, the Klein-Gordon equation and the Dirac equation can be considered as field equations. Each of them corresponds to Euler-Lagrange

*Email: vdevanathan@hotmail.com

equation of motion, that can be deduced from the corresponding Lagrangian. Each Lagrangian has a certain phase or gauge symmetry and is said to be invariant under the phase or gauge transformation. If the phase is independent of the space-time coordinates, then the invariance of the Lagrangian under the gauge transformation is said to be *global*. If the phase is dependent on the space-time coordinates, then the Lagrangian is no longer invariant but the invariance can be restored by including an additional term in the Lagrangian. The additional term is the interaction term due to *the gauge boson* in the Lagrangian. The invariance of the Lagrangian under a phase transformation which is dependent on the space-time coordinates is known as *the local gauge invariance*. In other words, a Lagrangian which is invariant under certain global gauge transformation can also be made to satisfy the same local gauge transformation by introducing an additional term corresponding to an interacting boson. Thus, the local gauge transformation dictates the interaction dynamics of the Lagrangian.

Let us consider the Dirac Lagrangian for the electron. It is invariant under the gauge transformation of the field function $\psi(\mathbf{x}) \rightarrow \psi(\mathbf{x})e^{i\alpha}$, where α is a real constant. If $\alpha(\mathbf{x})$ is a function of space-time coordinate $\mathbf{x}$, then the Lagrangian can be made to be invariant under local gauge transformation $\psi(\mathbf{x}) \rightarrow \psi(\mathbf{x})e^{i\alpha(\mathbf{x})}$ by introducing an additional term corresponding to an interaction with electromagnetic field A_μ. The additional field A_μ is called the *gauge field*, the quantum of the field is called the *gauge boson* and the underlying theory is called the *gauge theory*. In the present case, the gauge boson is the photon which has rest mass zero.

In the *Standard Model of Elementary Particles*, the fundamental particles are six leptons (e^-, ν_e), (μ^-, ν_μ), (τ, ν_τ), six quarks (u, d, s, c, b, t) and the corresponding anti-leptons and anti-quarks. The interaction between them takes place by exchange of gauge bosons. In the standard model, we consider only three types of interaction - electro-magnetic, strong and weak interactions - and the gravitational interaction is outside its purview. The electromagnetic interaction arises from exchange of photons, the strong interaction by exchange of gluons and the weak interaction by exchange of intermediate vector bosons. Photons and gluons have rest mass zero whereas the intermediate vector bosons W^+, W^-, Z^0 are massive with rest mass of about 80 GeV.

Of all the three interactions, the gauge theory of electromagnetic interaction is the simplest, since the Lagrangian is invariant under U(1) gauge transformation and the gauge boson is the photon with rest mass zero. Since the quarks are coloured objects with three different colours, the Lagrangian for the strong interaction has SU(3) gauge symmetry and the theory of strong interaction is known as *Quantum Chromodynamics*. The strong interaction arises from exchange of gluons which are once again particles of zero mass. But there is a problem in developing a gauge theory for weak interaction since the interaction arises from an exchange of intermediate vector bosons which are massive. The introduction of mass term in the Lagrangian spoils the symmetry of the Lagrangian. *The invariance of the Lagrangian under gauge transformation is an essential condition for developing a successful field theory* [3,4,5]. How to add mass to the gauge boson without violating the gauge symmetry of the Lagrangian? That was the vexing problem in 1964. Three different groups [6,7,8] were simultaneously but independently working on this problem and they came out with a similar solution.

If a massless gauge boson interacts with a complex scalar field, the lowest eigenstate of the interacting gauge boson with the complex scalar field is shifted from the symmetry axis of the Lagrangian. If the Lagrangian is re-written with reference to the lowest eigenstate, then the gauge boson acquires a mass. **The original Lagrangian has the symmetry but the same Lagrangian re-written with respect to the lowest eigenstate has lost the symmetry and this is known as *the spontaneously broken symmetry*.**

The complex scalar field that is postulated is known as *Higgs field* and it is a field of a scalar particle with imaginary mass, rather an energy field which pervades the entire universe. The massless gauge boson interacting with Higgs field acquires mass. This way of acquiring mass by the gauge boson is known as *the Higgs mechanism.*

The weak interaction Lagrangian is invariant under SU(2) gauge transformation but the Lagrangian of the unified electro-weak interaction is invariant under SU(2) × U(1) gauge transformation. In the electro-weak Lagrangian, the mass term of the fermion is to be dropped since it is not invariant under the combined SU(2) × U(1) gauge transformation but the mass of the fermion can be recovered by its interaction with Higgs field. In a similar way, all leptons and quarks acquire masses by interaction with Higgs field. **Thus the Higgs field which was first postulated to endow mass to the gauge boson without violating the gauge invariance of the Lagrangian has now assumed a greater role of endowing mass to all the particles.**

The Higgs field is just an hypothetical field that is thought to pervade the entire universe. Is there any experimental evidence for it? Yes, there is. The interacting Lagrangian which includes Higgs field not only supplies mass to the gauge boson and to the particles but also produces a massive scalar particle (Hiigs boson) in this process. The experimental detection of this Higgs boson will validate the postulate of Higgs field. That is why, the CERN has set up an international collaboration for detecting the Higgs boson and succeeded in its efforts after a lapse of 50 years.

Higgs boson can be considered as an excited quantal state of Higgs field and thus the experimental discovery of Higgs boson offers a strong support to the concept of Higgs field, pervading the entire universe.

2 Lagrangian and Euler-Lagrange Equation

Given the Lagrangian of a mechanical system

$$L = L(q_k(t), \dot{q}_k(t), t),$$

where $q_k(t)$ and $\dot{q}_k(t) = \frac{dq_k}{dt}$ with $k = 1, 2, \cdots, n$ denote the generalized coordinates and velocities which are functions of time t, the Action Integral $\mathscr{A}$ is given by

$$\mathscr{A} = \int_{t_1}^{t_2} L(q_k(t), \dot{q}_k(t), t) dt.$$

According to the variational principle[1], the action integral has a stationary value and the variation $\delta\mathscr{A}$ of $\mathscr{A}$ due to small variation in path with fixed end-points is identically zero. This leads to the Euler-Lagrange equation of motion [1,2].

$$\frac{\partial L}{\partial q_k} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} = 0, \qquad k = 1, 2, \ldots, n. \tag{1}$$

The Lagrangian formalism which has been developed for a discrete mechanical system can be extended to a field, described by a field function $\psi(\boldsymbol{x}, t)$ where $\boldsymbol{x}$ and t are continuous variables. For this, instead of the Lagrangian L, we need to work with the Lagrangian density $\mathscr{L}$.

$$\mathscr{L}\left(\psi_\rho, \frac{\partial \psi_\rho}{\partial x_\nu}, \mathbf{x}\right) \equiv \mathscr{L}\left(\psi_\rho, \psi_{\rho,\nu}, \mathbf{x}\right),$$

where, for brevity, a notation $\psi_{\rho,\nu}$ is used to denote $\frac{\partial \psi_\rho}{\partial x_\nu}$. The Lagrangian density is a function of field functions ψ_ρ, its first order derivatives and the space-time coordinates $(\mathbf{x} : x_0 = ct, x_1, x_2, x_3)$. It is important that $\mathscr{L}$ should not depend on second and higher order derivatives of ψ_ρ.

The Lagrangian and the action integral are given by

$$L = \int \mathscr{L}\left(\psi_\rho, \psi_{\rho,\nu}, \mathbf{x}\right) d^3x; \tag{2}$$

$$\mathscr{A} = \int_V \mathscr{L}\left(\psi_\rho, \psi_{\rho,\nu}, \mathbf{x}\right) d^4x; \tag{3}$$

where

$$d^4x = dx_0 dx_1 dx_2 dx_3 = c dt dx dy dz,$$

[1] Of the various paths available between the two given end points, the system chooses the path for which the action integral $\mathscr{A}$ is minimum. This is found by the *calculus of variations*.

and V is a certain domain in space-time, bounded by a hypersurface S. According to the principle of least action, the variation $\delta\mathscr{A}$ in $\mathscr{A}$ for arbitrary domains V is zero for the variations of $\psi_\rho(\mathbf{x})$. This leads to the following Euler-Lagrange equations, analogous to Eq. (1), for the Lagrangian density of fields.

$$\frac{\partial\mathscr{L}}{\partial\psi_\rho} - \sum_\nu \frac{\partial}{\partial x_\nu}\frac{\partial\mathscr{L}}{\partial\frac{\partial\psi_\rho}{\partial x_\nu}} = 0, \qquad \nu = 0,1,2,3, \qquad \rho = 1,2,\ldots,n. \tag{4}$$

These equations, in turn, lead to the field equations. Since $\mathscr{L}$ does not involve derivatives of ψ_ρ of order higher than the first, the resulting field equations will be utmost of second order.

Using the short-hand notation, the Euler-Lagrange Eq. (4) can be rewritten as

$$\frac{\partial\mathscr{L}}{\partial\psi_\rho} - \partial_\nu\frac{\partial\mathscr{L}}{\partial(\partial_\nu\psi_\rho)} = 0, \qquad \rho = 1,2,\ldots,n, \tag{5}$$

with the convention that a summation is to be made on the repeated indices.

The Schrödinger equation, the Klein-Gordon equation and the Dirac equation are the field equations that are on a par with the Euler-Lagrange equation (5). Given the Lagrangian densities,

$$\mathscr{L}_S = i\hbar\psi^*\dot{\psi} - \frac{\hbar}{2m}\boldsymbol{\nabla}\psi^*\cdot\boldsymbol{\nabla}\psi - V\psi^*\psi, \tag{6}$$

$$\mathscr{L}_{KG} = \frac{1}{2}\sum_\nu\left(\frac{\partial\psi}{\partial x^\nu}\right)^2 - \frac{1}{2}\mu^2\psi^2, \quad \text{where} \quad \mu = \frac{mc}{\hbar} \tag{7}$$

$$\mathscr{L}_D = \bar{\psi}(\mathbf{x})(i\gamma^\nu\partial_\nu - \mu)\psi(\mathbf{x}), \qquad \text{where} \quad \mu = \frac{mc}{\hbar} \tag{8}$$

we can obtain, using the Euler-Lagrange equation (5), the corresponding field equations.

$$\text{Schrodinger Eq.} \quad -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi - i\hbar\frac{\partial\psi}{\partial t} = 0. \tag{9}$$

$$\text{K.G. Eq.} \qquad (\Box + \mu^2)\psi(\mathbf{x}) = 0, \quad \Box = \frac{\partial}{\partial x^\nu}\frac{\partial}{\partial x_\nu} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2. \tag{10}$$

$$\text{Dirac Eq.} \qquad (i\gamma^\nu\partial_\nu - \mu)\psi(\mathbf{x}) = 0. \tag{11}$$

In natural units, $\hbar = c = 1$, $\mu = m$. Thus, we have shown that for each field equation, there is a corresponding Lagrangian density and that field equation is just the Euler-Langrage equation obtained with that Lagrangian density.

3 The Gauge invariance of the Lagrangian density

In classical mechanics, we deal with Lagrangians L but in field theory, we only work with Lagrangian densities $\mathscr{L}$. Since the present study is in Quantum Field Theory, let us change the nomenclature for simplicity and call hereafter the Lagrangian density $\mathscr{L}$, simply as Lagrangian.

The Lagrangian is invariant under a phase or gauge transformation of the field function. There are two types of gauge transformations *global gauge transformation* and *local gauge transformation*. In global gauge transformation, the gauge transformation is independent of space-time coordinates and in local gauge transformation, the gauge transformation is dependent on the space-time coordinates. The Lagrangian which is invariant under global gauge transformation is no longer found to be invariant under local gauge transformation but, in some cases, the invariance of the Lagrangian may be restored by adding an additional term corresponding to a gauge boson in the Lagrangian. Let me illustrate this point by giving a specific example.

3.1 The Global gauge transformation

Let us consider the Dirac Lagrangian (in natural units) for electron of rest mass m.

$$\mathscr{L} = \bar{\psi}(\mathbf{x})(i\gamma^\mu \partial_\mu - m)\psi(\mathbf{x}), \tag{12}$$

where $\gamma^\mu,\ \mu = 0, 1, 2, 3$ denote the four gamma matrices, $\bar{\psi} = \psi^\dagger \gamma^0$, $\mathbf{x} = x^0, x^1, x^2, x^3$ and $\partial_\mu = \frac{\partial}{\partial x^\mu}$. It can be easily checked that the Lagrangian $\mathscr{L}$ is invariant under the phase transformation

$$\psi(\mathbf{x}) \to e^{i\alpha}\psi(\mathbf{x}), \tag{13}$$

where α is a real constant. It follows that

$$\partial_\mu \psi \ \to \ e^{i\alpha}\partial_\mu \psi. \tag{14}$$
$$\bar{\psi} \ \to \ e^{-i\alpha}\bar{\psi}. \tag{15}$$

The phase transformation $U(\alpha) = e^{i\alpha}$, with a single parameter α running over all real numbers, forms a unitary Abelian group $U(1)$. It is called Abelian since the group multiplication is commutative.

$$U(\alpha_1)U(\alpha_2) = U(\alpha_2)U(\alpha_1).$$

3.2 The Local gauge transformation

Let us investigate what happens to the Lagrangian under local phase (gauge) transformation. In this case, α is a function of space-time coordinates $\mathbf{x}$. From Eq. (13), it follows

$$\begin{aligned} \psi(\mathbf{x}) &\longrightarrow \psi'(\mathbf{x}) = e^{i\alpha(\mathbf{X})}\psi(\mathbf{x}) && (16)\\ \partial_\mu\psi(\mathbf{x}) &\longrightarrow \partial_\mu\psi'(\mathbf{x}) = \partial_\mu\left(e^{i\alpha(\mathbf{X})}\psi(\mathbf{x})\right) \\ &= \left(\partial_\mu e^{i\alpha(\mathbf{X})}\right)\psi(\mathbf{x}) + e^{i\alpha(\mathbf{X})}\partial_\mu\psi(\mathbf{x}) \\ &= i\left(\partial_\mu\alpha(\mathbf{x})\right)e^{i\alpha(\mathbf{X})}\psi(\mathbf{x}) + e^{i\alpha(\mathbf{X})}\partial_\mu\psi(\mathbf{x}). && (17) \end{aligned}$$

For Fermions,

$$\begin{aligned} \mathscr{L}_{\psi'} &= \bar{\psi}'(i\gamma^\mu\partial_\mu - m)\psi' \\ &= -\bar{\psi}(\partial_\mu\alpha)\gamma^\mu\psi + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \\ &= -(\partial_\mu\alpha)\bar{\psi}\gamma^\mu\psi + \mathscr{L}_\psi. && (18) \end{aligned}$$

Equation (18) clearly indicates that the Dirac Lagrangian is not invariant under local phase transformation. This failure can be rectified by introducing new fields such that the modified Lagrangian has the local symmetry. The extra term on the right-hand side of Eq. (18) involves a factor $\partial_\mu\alpha$ which transforms like a four-vector. Introducing an additional term with a four-vector field A_μ in the Dirac Lagrangian, a modified Lagrangian

$$\mathscr{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + e\bar{\psi}\gamma^\mu\psi A_\mu, \tag{19}$$

can be obtained which has a local symmetry under the transformation

$$\psi \to \psi' = e^{i\alpha(\mathbf{X})}\psi, \qquad A_\mu \to A'_\mu. \tag{20}$$

What should be the transformation property of A_μ such that the modified Lagrangian $\mathscr{L}'$ is invariant under the local phase transformation? The modified Lagrangian $\mathscr{L}'$ is

$$\begin{aligned} \mathscr{L}' &= \bar{\psi}'(i\gamma^\mu\partial_\mu - m)\psi' + e\bar{\psi}'\gamma^\mu\psi' A'_\mu \\ &= \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + e\left(A'_\mu - \frac{1}{e}\partial_\mu\alpha\right)\bar{\psi}\gamma^\mu\psi. && (21) \end{aligned}$$

$\mathscr{L}'$ is invariant if the newly introduced four-vector field A_μ transforms as

$$A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\alpha. \tag{22}$$

Thus, we find that the Lagrangian $\mathscr{L}$ which is invariant under global gauge transformation is not invariant under local gauge transformation but the modified Lagrangian $\mathscr{L}'$ which includes an interaction term with a gauge field A_μ is invariant under local gauge transformation. Thus the requirement of the local gauge invariance dictates the interaction dynamics of the Lagrangian.

Since the origin of the interaction term can be traced to the introduction of a new field to make the Dirac Lagrangian invariant under a local gauge transformation, the newly introduced field has come to be known as *gauge field*, its field quantum as *gauge Boson* and the underlying theory as *gauge theory*.

4 Gauge theory of Electro-magnetic interaction

Now one can easily identify that the newly introduced field A_μ is none other than the electromagnetic field; interaction with which makes the Dirac Lagrangian of the electron invariant under the same local transformation. Adding the free field Lagrangian of the electromagnetic field[2], we obtain the full Lagrangian for the combined field of ψ and A_μ.

$$\mathscr{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + e\bar{\psi}\gamma^\mu\psi A_\mu. \tag{23}$$

This is the Lagrangian that is used in field theory for the study of Quantum Electrodynamics (QED). It may be observed that the full Lagrangian (23) consists of an interaction term

$$\mathscr{L}_{\text{int}} = e\bar{\psi}\gamma^\mu\psi A_\mu,$$

besides the free-field Lagrangians of Dirac and e.m. fields.
Please note that the Lagrangian (23) does not contain any mass term for the gauge boson A_μ since the photon is massless.

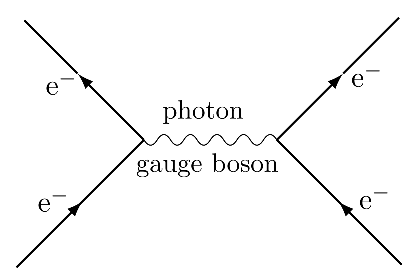

Fig. 1: Electron-electron interaction

If one includes a mass term for the gauge field, the Lagrangian will no longer be invariant under gauge transformation. *The invariance of Lagrangian under gauge*

[2]Since the electromagnetic field strength tensor $F_{\mu\nu}$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

is gauge invariant, the total Lagrangian $\mathscr{L}$ remains gauge invariant.

transformation is an essential condition to be fulfilled for developing any successful field theory [3,4,5].

4.1 Gauge covariant derivative

Let us define a *gauge covariant derivative*[3] D_μ.

$$D_\mu = \partial_\mu - ieA_\mu, \tag{24}$$

such that the complete Lagrangian for QED (Eq. (23)) can be written in the form

$$\mathscr{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{25}$$

The same technique can be used for any other field with global symmetry. Let us illustrate it for the charged scalar field ϕ. Any complex scalar field ϕ which has a global phase symmetry for the transformation

$$\phi \to e^{i\alpha}\phi$$

will also acquire a local phase symmetry with the substitution

$$\partial_\mu \to D_\mu; \qquad A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\alpha.$$

Thus we can obtain the Lagrangian for the complex scalar field ϕ interacting with the electromagnetic field A_μ as

$$\mathscr{L} = (D_\mu\phi)^\dagger(D_\mu\phi) - m^2\phi^\dagger\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{26}$$

The Lagrangian (Eq. 26) contains the mass term for the complex scalar field but not for the gauge boson.

The e.m. interaction is a large range interaction since the exchanged gauge boson is massless, whereas the weak interaction is of very short range since the exchanged gauge boson W^- is massive.

The Lagrangian will no longer be invariant under the gauge transformation, if a mass term for the gauge boson is added, but the invariance of Lagrangian under

[3]The gauge covariant derivative of ψ, D_μ transforms in the same way as ψ.

$$\psi(\mathbf{x}) \to e^{i\alpha(\mathbf{x})}\psi(\mathbf{x}); \qquad D_\mu\psi(\mathbf{x}) \to e^{i\alpha(\mathbf{x})}D_\mu\psi(\mathbf{x}).$$

gauge transformation is an essential condition for developing a successful gauge theory.

How to add a mass term for the gauge boson without violating the invariance of the Lagrangian under gauge transformation? This was the problem that was faced in nineteen sixties. Three different groups [6,7,8] working independently came out with a similar solution in 1964.

Allow the gauge boson to interact with a hypothetical complex scalar field with imaginary mass (an energy field), which is now called the Higgs field, named after one of the proponents. The Higgs field is postulated to pervade the entire universe. Find the lowest eigenstate of the gauge boson interacting with the Higgs field and it is found to be shifted from the symmetry axis of the Lagrangian. If the Lagrangian is re-written with reference to the lowest eigenstate, the symmetry is lost and it is known as *the spontaneous broken symmetry*. In the re-written Lagrangian, a mass term for the gauge boson appears. This is done without violating the gauge symmetry of the original Lagrangian. This way of generating the mass for the gauge boson is known as *the Higgs mechanism*.

4.2 Spontaneous Symmetry Breaking and Higgs Mechanism

Let us start with the QED Lagrangian (26) for a charged scalar field of particle with mass μ and include the self interaction term $\lambda(\phi^*\phi)^2$ for the scalar field [2].

$$\mathscr{L} = (\partial^\mu + ieA^\mu)\phi^*(\partial_\mu - ieA_\mu)\phi - V - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{27}$$

with

$$V = \mu^2\phi^*\phi + \lambda(\phi^*\phi)^2.$$

This Lagrangian (27) is invariant under a U(1) local gauge transformation.

$$\phi \to e^{i\alpha(\mathbf{X})}\phi.$$

It corresponds to a scalar field ϕ of mass μ and a gauge field A_μ which is massless. By treating μ^2 and λ as parameters and taking $\mu^2 < 0$ and $\lambda > 0$, we will be able to generate a mass for the gauge boson A^μ by the method of spontaneous symmetry breaking. This way of generating a mass for gauge boson is known as the Higgs mechanism.

Minimizing the potential

$$V = \mu^2\phi^*\phi + \lambda(\phi^*\phi)^2, \qquad \frac{dV}{d\phi} = \mu^2\phi^* + 2\lambda(\phi^*\phi)\phi^* = 0,$$

we get

$$|\phi|^2 = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}, \quad v = \sqrt{-\frac{\mu^2}{\lambda}}.$$

This corresponds to the vacuum expectation value

$$\langle\phi\rangle = \frac{v}{\sqrt{2}} = \sqrt{-\frac{\mu^2}{2\lambda}}. \tag{28}$$

Only the magnitude is obtained but its phase is arbitrary. There are many ground states located on a circle of radius v with the same energy as shown in Fig. 2(b) and the system can be in anyone of the ground states.

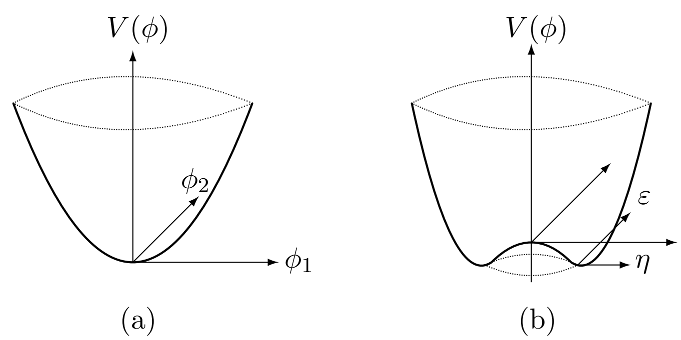

Figure 2: The shape of $V(\phi)$ for a complex scalar field with $\lambda > 0$ and (a) $\mu^2 > 0$ and (b) $\mu^2 < 0$.

We can now expand the complex scalar field $\phi(x)$ in terms of two real fields $\eta(x)$ and $\varepsilon(x)$ which have zero expectation values at the ground state.

$$\phi(x) = \sqrt{\frac{1}{2}}[v + \eta(x) + i\varepsilon(x)]. \tag{29}$$

This yields

$$\begin{aligned} \phi^*\phi &= \frac{1}{2}\{(v+\eta)^2 + \varepsilon^2\}. && (30) \\ V &= \mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2, \quad \text{with} \quad \mu^2 = -v^2\lambda \\ &= \frac{\lambda}{4}\left\{(v+\eta)^2 + \varepsilon^2\right\}\left\{-2v^2 + (v+\eta)^2 + \varepsilon^2\right\} \\ &= \frac{\lambda}{4}\left\{(\eta^2 + \varepsilon^2 + 2v\eta)^2 - v^4\right\}. && (31) \end{aligned}$$

Expanding the complex scalar field ϕ in terms of the real fields η and ε that correspond to the ground state of the system

$$\phi(x) = \sqrt{\frac{1}{2}}\{v + \eta(x) + i\varepsilon(x)\},$$

we can rewrite the Lagrangian $\mathscr{L}$ in terms of the new fields $\eta(x)$ and $\varepsilon(x)$.

The first term in the Lagrangian (27) yields

$$\frac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) + \frac{1}{2}(\partial^\mu \varepsilon)(\partial_\mu \varepsilon) + e^2 A^\mu A_\mu \phi^* \phi.$$

Using the expressions (30) and (31), deduced earlier for $\phi^*\phi$ and V, we rewrite the Lagrangian (27) in terms of the new fields η and ε.

$$\begin{aligned}\mathscr{L}' &= \frac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) + \frac{1}{2}(\partial^\mu \varepsilon)(\partial_\mu \varepsilon) + \frac{1}{2}e^2 v^2 A^\mu A_\mu \\ &\quad - v^2 \lambda \eta^2 - ev A_\mu \partial^\mu \varepsilon - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \text{interaction terms.}\end{aligned} \tag{32}$$

The Lagrangian $\mathscr{L}'$ exhibits a particle spectrum which consists of a massive gauge vector boson A_μ, a massive scalar η, a massless Goldstone boson ε. From an inspection of the Lagrangian (32), we can write down the masses of the particles.

$$m_A = ev, \qquad m_\eta = \sqrt{2\lambda v^2}, \qquad m_\varepsilon = 0.$$

In this way, we have dynamically generated a mass for the gauge boson A_μ but along with it, we have also a massless Goldstone boson ε. The Lagrangians $\mathscr{L}$ and $\mathscr{L}'$ are equivalent and the number of degrees of freedom cannot change by this transformation. In $\mathscr{L}$, we have a complex scalar field with mass which accounts for two degrees of freedom and a massless gauge boson A_μ with two degree of freedom corresponding to transverse polarizations. Thus, in total, we have four degrees of freedom in the Lagrangian $\mathscr{L}$. On the other hand, in $\mathscr{L}'$, the gauge boson A_μ has acquired a mass and so it can have longitudinal polarization in addition to two transverse polarizations. There are also two scalar particles, one with mass (η) and another without mass (ε) (known as Goldstone boson). Thus, we have, in total, five degrees of freedom in the Lagrangian $\mathscr{L}'$. The Goldstone boson, which is spurious, has to be eliminated. The presence of off-diagonal term $evA_\mu\partial^\mu\varepsilon$ in $\mathscr{L}'$ requires some attention. Is it possible to eliminate the field ε by choosing some particular gauge transformation? Yes, it is, by choosing a slightly modified gauge transformation.

4.2.1 The unitary gauge

We get a clue from the expansion of the field ϕ in terms of the fields η and ε which define the ground state.

$$\phi = \sqrt{\frac{1}{2}}(v + \eta + i\varepsilon).$$

This is the lowest order in ε but instead of this, we shall include higher orders in ε and write [2]

$$\phi = \sqrt{\frac{1}{2}}(v+\eta)e^{i\varepsilon/v}.$$

This suggests that we should substitute a different set of real fields h, θ, A_μ.

$$\phi \longrightarrow \phi' = \sqrt{\frac{1}{2}}(v+h(x))e^{i\theta(x)/v}; \tag{33}$$

$$A_\mu \longrightarrow A'_\mu = A_\mu + \frac{1}{ev}\partial_\mu\theta \tag{34}$$

This is a particular choice of gauge, known as the unitary gauge, that makes the Lagrangian independent of the field θ.

$$\mathscr{L}'' = (\partial^\mu + ieA'^\mu)\phi'^*(\partial_\mu - ieA'_\mu)\phi' - V - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{35}$$

where

$$\begin{aligned} V &= \mu^2\phi'^*\phi' + \lambda(\phi'^*\phi')^2, \quad \text{with} \quad \mu^2 = -v^2\lambda \\ &= \frac{\lambda}{4}\left(h^4 + 4vh^3 + 4v^2h^2 - v^4\right). \end{aligned} \tag{36}$$

The first term in (35) yields

$$\begin{aligned} &(\partial^\mu + ieA'^\mu)\phi'^*(\partial_\mu - ieA'_\mu)\phi' \\ &= \frac{1}{2}\partial^\mu h\,\partial_\mu h + \frac{1}{2}e^2v^2A^\mu A_\mu + \frac{1}{2}e^2h^2A^\mu A_\mu + e^2vhA^\mu A_\mu. \end{aligned} \tag{37}$$

Substituting (36) and (37) into the Lagrangian (35) and rearranging, we get

$$\begin{aligned} \mathscr{L}'' &= \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2h^2 - \frac{1}{2}e^2v^2A_\mu^2 - \lambda vh^3 - \frac{1}{4}\lambda h^4 \\ &\quad + \frac{1}{2}e^2h^2A_\mu^2 + ve^2hA_\mu^2 + \frac{1}{4}\lambda v^4 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}. \end{aligned} \tag{38}$$

We have successfully eliminated the Goldstone boson which is spurious. The Lagrangian (38) just describes two interacting massive particles, the massive gauge vector boson A_μ and a massive scalar boson h which is called the Higgs boson.

$$m_A = ev, \qquad m_h = \sqrt{2\lambda v^2}.$$

The unwanted massless Goldstone boson has been used up to provide the longitudinal polarization for the massive gauss vector boson A_μ. Since the above study offers a method of producing massive gauge vector boson by eliminating the spurious Goldstone boson, this method is also known as *the Higgs mechanism*.

5 Gauge Theory of Weak Interaction

Just as the e.m. interaction is explained by the exchange of photons, the weak interaction can also be explained by exchange of gauge bosons. In this case, the gauge bosons are intermediate vector bosons which are massive with mass in the order of 80 GeV/c^2.

For illustrative purpose, let us consider the beta decay of the neutron.

$$n \longrightarrow p + e^- + \bar{\nu}_e.$$

This decay can be represented by the the Feynman diagram as shown in Fig.3. The neutron emits an intermediate vector boson W^- and transforms into a proton and the emitted boson W^- decays into e^- and $\bar{\nu}_e$. In Feynman diagrams, the particles travel forward in time while the anti-particles travel backward in time.

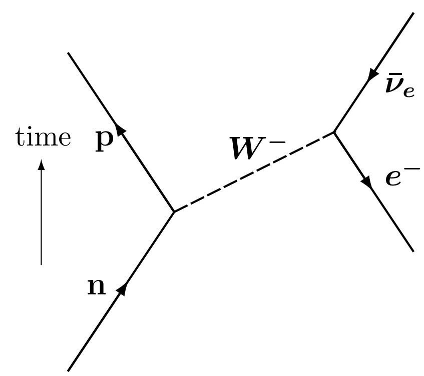

Fig. 3: Feynman diagram for beta decay

The e.m. interaction is a large range interaction since the exchanged gauge boson is massless, whereas the weak interaction is of very short range since the exchanged gauge boson W^- is massive.

The Lagrangian will no longer be invariant under the gauge transformation, if a mass term for the gauge boson is added, but the invariance of Lagrangian under gauge transformation is an essential condition for developing a successful gauge theory.

How to add a mass term for the gauge boson without violating the invariance of the Lagrangian under gauge transformation? This was the problem that was faced in nineteen sixties. Three different groups [6,7,8] working independently came out with a similar solution in 1964.

Allow the gauge boson to interact with a hypothetical complex scalar field with imaginary mass (an energy field), which is now called the Higgs field, named after one of the proponents. The Higgs field is postulated to pervade the entire universe. Find the lowest eigenstate of the gauge boson interacting with the Higgs field and it is found to be shifted from the symmetry axis of the Lagrangian. If the Lagrangian is re-written with reference to the lowest eigenstate, the symmetry is lost and it is known as *the spontaneous broken symmetry*. In the re-written Lagrangian, a mass term for the gauge boson appears. This is done without violating the gauge symmetry of the original Lagrangian. This way of generating the mass for the gauge boson is known as *the Higgs mechanism*.

Proton and neutron can be considered as two charge states of a nucleon and accordingly the nucleon field is to be treated as a doublet in the iso-spin space of the nucleon.

$$\Psi_N = \begin{bmatrix} \psi_p \\ \psi_n \end{bmatrix}. \tag{39}$$

The Lagrangian density of the nucleon is invariant under the global gauge transformation $U = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\tau}/2}$, where the components of $\boldsymbol{\tau}$ are the Pauli iso-spin matrices

$$\tau_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \tau_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \tau_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{40}$$

and the components of α are $\alpha_x, \alpha_y, \alpha_z$ which are constants, independent of the space-time coordinates. Then

$$\Psi_N \longrightarrow \Psi'_N = U\Psi_N = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\tau}/2}\Psi_N \tag{41}$$

$$\overline{\Psi}_N \longrightarrow \overline{\Psi}'_N = \overline{\Psi}_N U^\dagger = \overline{\Psi}_N e^{-i\boldsymbol{\alpha}\cdot\boldsymbol{\tau}/2}. \tag{42}$$

Since $\det U = 1$, this corresponds to SU(2) unitary transformation and it is said to be non-Abelian since the generators τ_x, τ_y, τ_z do not commute.

In order to make the Lagrangian invariant under local gauge transformation, we need to introduce three massless gauge bosons, which will acquire mass by interaction with the Higgs field, invoking the concept of spontaneous symmetry breaking and Higgs mechanism.

In the present case, we need to consider a SU(2) doublet structure for the complex scalar field ϕ with four components $\phi_1, \phi_2, \phi_3, \phi_4$.

$$\phi = \begin{bmatrix} \phi_\alpha \\ \phi_\beta \end{bmatrix} = \sqrt{\frac{1}{2}} \begin{bmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{bmatrix}. \tag{43}$$

It can be easily checked that the Lagrangian $\mathscr{L}$

$$\mathscr{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2, \tag{44}$$

is invariant under global SU(2) phase transformation.

$$\phi \to \phi' = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\tau}/2}\phi. \tag{45}$$

To make it invariant under local gauge transformation with $\boldsymbol{\alpha}(\mathbf{x})$, we need to replace the derivative ∂_μ with covariant derivative

$$D_\mu = \partial_\mu + \frac{1}{2} ig\tau_a W_\mu^a, \tag{46}$$

invoking three gauge vector fields W_μ^a with $a = 1, 2, 3$. The strength of the SU(2) coupling to the Gauge fields is denoted by g.

The lowest eigenstates of the scalar fields interacting with the gauge fields W^a are shifted from the symmetry axis of the Lagrangian and if the Lagrangian is re-written with reference to one of the lowest eigenstates, then the symmetry of the Lagrangian is spontaneously broken. The three gauge vector bosons W^a acquire masses by absorbing the three scalar fields and the fourth scalar field yields the massive Higgs boson. The experimental detection of the massive Higgs boson is the only way by which the postulate of Higgs field (scalar energy field), pervading the entire universe, can be verified.

6 Electroweak Interaction

The unified gauge theory of elctromagnetic and weak interactions was successfully developed by Weinberg and Abdus Salam [9,10], invoking invariance under SU(2) × U(1) gauge transformation. The standard model of elementary particles admits three generations of leptons and quarks.

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} u \\ d \end{pmatrix}; \qquad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}; \qquad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}.$$

For studying their weak interactions, they are grouped together by defining weak isospin T and weak hypercharge Y quantum numbers, such that the charge of the particle is given by

$$Q = T_3 + \frac{Y}{2}. \tag{47}$$

In Table 1, the weak isospin T and weak hypercharge Y quantum numbers are given only for the firat generation of leptons and quarks. Similar quantum numbers are given for the second and third generations of leptons and quarks. According to the standard model of elementary particles, the neutrino is left-handed with zero rest mass. Since the electron has a finite mass, it can have both right-handed and left-handed components. As shown in Table 1, the electron neutrino and the left-handed electron are paired together to form a doublet with weak iso-spin $T = \frac{1}{2}$ and weak hypercharge $Y = -1$ but different iso-spin projections $T_3 = +\frac{1}{2}$ and $T_3 = -\frac{1}{2}$. The right chiral electron is a singlet with weak iso-spin $T = 0$ and weak hypercharge $Y = -2$.

$$\psi_L = \begin{bmatrix} \nu_e \\ e_L^- \end{bmatrix} \quad \text{with } T = \tfrac{1}{2},\ Y = -1; \tag{48}$$

$$\psi_R = e_R^- \quad \text{with } T = 0,\ Y = -2. \tag{49}$$

Table 1: Weak isospin T and weak hypercharge Y quantum numbers of the first generation of leptons and quarks with charge $Q = T + \frac{Y}{2}$

Lepton	T	T_3	Q	Y	Quark	T	T_3	Q	Y
ν_e	$\frac{1}{2}$	$\frac{1}{2}$	0	-1	u_L	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$
e_L^-	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	d_L	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$
					u_R	0	0	$\frac{2}{3}$	$\frac{4}{3}$
e_R^-	0	0	-1	-2	d_R	0	0	$-\frac{1}{3}$	$-\frac{2}{3}$

They are subjected to gauge transformation $SU(2) \times U(1)$ as shown below:

$$\psi_L \longrightarrow \psi'_L = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{T}+i\beta(Y/2)}\psi_L; \tag{50}$$

$$\psi_R \longrightarrow \psi'_R = e^{i\beta(Y/2)}\psi_R. \tag{51}$$

For global phase invariance, the quantities $\boldsymbol{\alpha}$ and β will be independent of space time coordinates and for local phase invariance, $\boldsymbol{\alpha}(\mathbf{x})$ and $\beta(\mathbf{x})$ will be dependent on the space-time coordinates $\mathbf{x}$.

In this scheme, the Dirac Lagrangian $\mathscr{L}$ of the electron-neutrino pair is invariant under global SU(2)×U(1) gauge transformation, only if the mass term of the electron is excluded from the Lagrangian as shown below.

$$\mathscr{L} = \overline{\psi}_L\gamma^\mu(i\partial_\mu)\psi_L + \overline{e}_R\gamma^\mu(i\partial_\mu)e_R,$$

In order to make the Lagrangian invariant under local gauge transformation, we need to introduce four vector gauge bosons W_1, W_2, W_3 and B in order to make the derivative ∂_μ to correspond to a covariant derivative D_μ.

$$(i\partial_\mu)_L \to (iD_\mu)_L = (i\partial_\mu)_L - g\boldsymbol{T}\cdot\boldsymbol{W} - g'\tfrac{1}{2}YB_\mu, \text{with } Y = -1; \tag{52}$$

$$(i\partial_\mu)_R \to (iD_\mu)_R = (i\partial_\mu)_R - g'\tfrac{1}{2}YB_\mu, \qquad \text{with } Y = -2. \tag{53}$$

Thus, we obtain the $SU(2) \times U(1)$ local gauge invariant Lagrangian of the electron-neutrino pair, by substituting $T = \frac{1}{2}\tau$ and the respective Y values.

$$\begin{aligned}\mathscr{L}_1 &= \overline{\psi}_L\gamma^\mu\left(i\partial_\mu - \tfrac{1}{2}g\boldsymbol{\tau}\cdot\boldsymbol{W} + \tfrac{1}{2}g'B_\mu\right)\psi_L + \overline{e}_R\gamma^\mu\left(i\partial_\mu + g'B_\mu\right)e_R \\ &\quad -\frac{1}{4}\boldsymbol{W}_{\mu\nu}\cdot\boldsymbol{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}.\end{aligned} \tag{54}$$

The last two terms in Eq. (54) represent the kinetic energies and self-coupling of the $\boldsymbol{W}_\mu$ fields and the kinetic energy of the B_μ field.

$$\boldsymbol{W}_{\mu\nu} = \partial_\mu \boldsymbol{W}_\nu - \partial_\nu \boldsymbol{W}_\mu - g\boldsymbol{W}_\mu \times \boldsymbol{W}_\nu; \tag{55}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{56}$$

Note that the Lagrangian $\mathscr{L}_1$ corresponds to massless fermions and massless gauge bosons. To generate the masses for the gauge bosons $\boldsymbol{W}_1, \boldsymbol{W}_2, \boldsymbol{W}_3$ and $\boldsymbol{B}$ in a gauge invariant way, we invoke the Higgs field and the Higgs mechanism of spontaneous symmetry breaking.

A suitable linear combinations of these gauge bosons yield the three massive vector bosons $\boldsymbol{W}^+, \boldsymbol{W}^-$ and $\boldsymbol{Z}^0$, associated with weak interaction and the massless photon $\boldsymbol{A}$, associated with electromagnetic interaction. The same Higgs doublet of scalar fields is used to generate lepton masses by spontaneous symmetry breaking but a slightly modified Higgs doublet of scalar fields is used to generate quark masses. Thus, all the particles including gauge bosons acquire mass by interaction with Higgs scalar fields by a process known as Higgs mechanism.

7 An Amazing Analogy

God loves symmetry and most of His creations have symmetry in their physical structure. Let us consider the structure of a human being. There is a left-right symmetry about the median line. Two hands, two legs, two eyes, two ears - all are symmetrically placed. In case of single organs such as head, nose and mouth, they are located on the median line. Even the internal organs, lungs and kidneys, exhibit such a symmetry. But, in the case of heart, there is only one but is slightly displaced to the left of the median line. The lowest eigenstate of a human being is the heart-beat and if it stops, the man is dead and gone. If one describes the features of the man with respect to the location of his lowest eigenstate (the heart-beat), the symmetry is lost and it is analogous to the spontaneously broken symmetry.

Let us continue the analogy and consider the human being to arise from the interaction of the human body with its soul.

Human Body + Soul = Human Being

However great and eminent a person may be, when you hear the sad news of his death, the first question that is asked is: "When the body is expected?" Then we pray for his soul and convey the condolences to his family as "May his soul rest in peace!"

What is the soul? It is an abstraction of life and it is a source of energy like Higgs field. Without life, nothing will grow and acquire a mass. Just as the matter fields and gauge fields acquire mass by interaction with Higgs field, the living beings acquire mass by interaction with soul. Just as Higgs boson serves as an evidence for the Higgs field, the growth of a living being serves for the concept of soul (life).

8 Concluding Remarks

The invariance of Lagrangian under gauge transformation is a necessary condition for developing a gauge theory of elementary particles and their interactions. The gauge theory of electro-magnetic interaction is highly successful since in this case the gauge boson (photon) is massless. Adding mass to the gauge boson spoils the invariance of Lagrangian under gauge transformation and this has led to the postulate of Higgs field (an energy field which is a scalar field with imaginary mass), the interaction with which shifts the lowest eigenstate of the interacting gauge boson to a location away from the symmetry axis of the Lagrangian. Although the original Lagrangian obeys the U(1) symmetry under gauge transformation, the Lagrangian written with respect to the lowest eigenstate does not reveal the gauge symmetry but allows the mass term for the gauge boson. This is known as the spontaneously broken symmetry and this way of allowing the gauge boson to acquire a mass is known as the Higgs mechanism. This has enabled the development of gauge theory of weak interaction with massive gauge bosons, obeying SU(2) gauge symmetry.

The unification of electromagnetic and weak interaction is made by invoking $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge symmetry of the Lagrangian and this does not permit the mass terms for fermions and leptons in the Lagrangian but the mass of fermions and leptons can be generated, using the same Higgs mechanism, by allowing them to interact with Higgs field.

Thus the Higgs field serves as a source of endowing all the particles with mass and, in this process, a massive scalar boson known as Higgs boson is emitted. The experimental detection of Higgs boson with a mass of about 126 $\mathrm{GeV/c^2}$ in the Large Hadron Collider (LHC) experiment at CERN in the year 2013 justifies the postulate of Higgs field pervading the entire universe. The discovery of Higgs boson is the crowning glory for the concept of Higgs field and Higgs mechanism and two of the originators of this idea, Peter W. Higgs and Francis Englert were jointly awarded the Nobel Prize in Physics for the year 2013.

Acknowledgement

I wish to convey my thanks to the Fellows of the Academy for extending their cooperation in initiating the E-Journal of the Academy which will help in dissemination of knowledge among the various research groups.

References

1. V. Devanathan, *Relativistic Quantum Mechanics and Quantum Field Theory*, Narosa Publishing House, New Delhi (2011); Alpha Science International, U.K. (2018)
2. V. Devanathan, *Textbook of Relativistic Quantum Physics*, Narosa Publishing House, New Delhi (2018); Alpha Science International, U.K. (2018)
3. F. Halzen and A. D. Martin, *Quarks and Leptons*, John Wiley (1984)
4. F. Mandl and G. Shaw, *Quantum Field Theory*, John Wiley (1984)
5. Gerard t. Hooft, *Nuclear Physics* **B 33**, 173-177 (1971); **B 35**, 167-448 (1971)
6. F. Englert and R. Brout, *Phys. Rev. Lett.* **13**, 321-323 (1964)
7. Peter W. Higgs, *Phys. Rev. Lett¿* **13**, 508-509 (1964)
8. S. Guralink, C. R. Hagen and T. W. B. Kibbs, *Phys. Rev. Lett.* **13**, 585-587 (1964)
9. S. Weinberg, *Phys. Rev. Lett.***27**, 1264 (1967)
10. Abdus Salam, *Proceedings of the 8th Nobel Symposium* (Ed. Svartholm, Almqvist and Wiksell), Stockhom (1968)