Issues of Confinement in QCD

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Received 20 August 2019

Abstract: The issues of confinement in QCD are reviewed within the frame work of quantum gauge field theory. Using 'cluster property' and BRST charge, the confining potential and observables are discussed. Quarks and gluons do not have asymptotic states while color singlet (gauge invariant) combinations have such states, implying that quarks and gluons are confined and not observable while baryons are observable.

1 Introduction

Gauge quantum field theories are the only quantum field theories relevant to particle physics. So it will be of physical importance to analyse the structure of these theories *without relying on perturbation theory.* This is particularly useful to address the issue of confinement of quarks (and gluons) in QCD since the confining regime is in the infra-red region where the QCD coupling is large so that the perturbative methods cannot be reliably employed. While the ultra violet regime involving short distance behaviour (high momentum) has small coupling strength, perturbative methods operate giving 'asymptotic freedom'. Asymptotic freedom in non-Abelian gauge theories [1], in particular in QCD, have been well established [2]. On the other hand, the long distance region characterized by large coupling strength is essentially non-perturbative. The confinement of quarks has to be addressed in this non-perturbative regime. Quantization of non-Abelian theories with large coupling

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strength suffered from gauge fixing procedure, known as Gribov ambiguity [3]. It is possible to avoid this difficulty by employing 'background gauge' [4].

The first assumption we make here is that in the confining regime, the QCD coupling is a constant, as suggested by Gribov [5].

Gauge quantum field theories have different properties from ordinary field theories. An example is the Abelian gauge quantum field theory in which the indefinite metric in the definition of scalar product plays crucial role. In non-Abelian gauge quantum field theories, the 'cluster property' does not necessarily hold, although such a property holds good on Abelian gauge quantum field theory.

The definition of physical space $\mathcal{V}_{Phys} \subset \mathcal{V}_{Total}$ such that the norm in $\mathcal{V}_{Phys}$ is positive semi-definite, that is, $(\phi, \phi) \geq 0$, $\phi \in \mathcal{V}_{Phys}$ is another distinguishing property of gauge quantum field theory. As the matrix elements between two physical states $\phi_1, \phi_2; \in \mathcal{V}_{Phys}$ do not change by adding to ϕ_1 and/or to ϕ_2 states $\chi \in \mathcal{V}_{Phys}$ with vanishing norm ($(\chi, \chi) = 0$), as these are also orthogonal to ϕ_1, ϕ_2 ($|(\phi, \chi)| \leq |(\phi, \phi)|^{\frac{1}{2}}\ |(\chi, \chi)|^{\frac{1}{2}}$ by Schwarz inequality, as $(\chi, \chi) = 0$, it follows $(\phi, \chi) = 0$), it is convenient to characterize the physical state corresponding to ϕ by the equivalence class $[\phi]$. The quotient $\mathcal{V}_{Phys} = \mathcal{V}_{Phys}/\mathcal{V}_0$, $\mathcal{V}_0 = \{\chi \in \mathcal{V}_{Phys}; (\chi, \chi) = 0\}$ will be called the space of physical states in which the scalar product is positive and definite.

We now consider observability condition in general. In a local gauge quantum field theory, with local symmetry group G unbroken, its generators Q^i commute with all the observables. A necessary condition for an operator A to describe an observable is $(\phi, [Q^i, A]\phi) = 0$. Consequently, in the Abelian gauge theory, Q corresponds to electric charge and so $(\psi_i, Q^i\psi_i) = q^i(\psi_i, \psi_i)$ is an observable. However, for QCD, we know $[Q^a, Q^b] = if^{abc}Q^c$ and so color charges cannot be observed. A deeper issue is whether a non-Abelian gauge quantum field theory has asymptotic particle like states with non-vanishing color. Such non-perturbative characteristic questions can be addressed now. The non observability of quarks means that quarks are associated with a basic set of fields $\psi_i(x)$ but no particle like asymptotic states exist with quark quantum numbers. The validity of the 'cluster property' becomes important in the existence of the asymptotic limit of a field operator. The failure of the 'cluster property' for the quark fields $\psi_i(x)$ is strictly related to the fact that the states $\psi_i|0\rangle$ do not have an asymptotic limit belonging to $\mathcal{V}_{Phys}$. So the question of a mechanism of confinement is the failure of the 'cluster property' of gauge quantum field theory [6].

In this review, two aspects of confinement in QCD will be focused. (1) The confining potential, such as linear potential, is plausible within the general frame work. (2) The observables in QCD in general will be gauge invariant. This implies that the (free) quarks can not be observed; quarks are confined while proton and neutron will be observed.

2 Quantum Yang-Mills theory

QCD is an $SU(3)_c$ unbroken gauge theory whose classical Lagrangian density is

$$\mathcal{L} \;=\; -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \bar{\psi}\gamma^\mu(i\partial_\mu + A^a_\mu t^a)\psi, \tag{1}$$

with $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$ and t^a's are generators of $SU(3)$. The classical equation of motion is

$$D^{ab}_\mu F^{\mu\nu b} \;=\; -gj^{\nu a}, \tag{2}$$

where $D^{ab}_\mu = \partial_\mu \delta^{ab} + gf^{acb}A^c_\mu$. By rewriting the classical equation of motion as

$$\partial_\mu F^{\mu\nu a} \;=\; -g\{j^{\nu a} + f^{acb}A^c_\mu F^{\mu\nu b}\} \;\equiv -gJ^{\nu a}, \tag{3}$$

we have a current $J^{\nu a} = j^{\nu a} + f^{acb}A^c_\mu F^{\mu\nu b}$ which is ordinarily conserved, i.e., $\partial_\nu J^{\nu a} = 0$. $J^{\nu a}$ contains a piece $f^{acb}A^c_\mu F^{\mu\nu b}$, a contribution from the gauge fields. Such a piece is absent in electromagnetic theory. The equation of motion from the *quantum YM theory* [7,8] is

$$\begin{aligned} D^{\mu\ ab}F^b_{\mu\nu} \;&=\; \partial_\nu B^a - gj^a_\nu - ig(\partial_\nu \bar{c} \times c)^a, \\ &=\; \partial_\nu B^a - gj^a_\nu - igf^{abc}\partial_\nu \bar{c}^b c^c, \end{aligned} \tag{4}$$

where c's are the Faddeev - Popov ghost fields and B^a's are the Lagrange multiplier fields in the gauge fixing part of the Lagrangian density and satisfy

$$D^{\mu\ ab}(\partial_\mu B^b) \;=\; igf^{abc}\partial_\mu \bar{c}^b\ (D^{\mu cd}c^d). \tag{5}$$

The BTST transformations

$$\begin{aligned} \delta A^a_\mu \;&=\; [iQ_B, A^a_\mu] \;=\; D^{ab}_\mu c^b, \\ \delta\psi^\alpha \;&=\; [iQ_B, \psi^\alpha] \;=\; igc^a(t^a)^\alpha_\beta \psi^\beta. \\ \delta B^a \;&=\; [iQ_B, B^a] \;=\; 0, \\ \delta c^a \;&=\; [iQ_B, c^a] \;=\; -\frac{g}{2}f^{abc}c^b c^c, \\ \delta \bar{c}^a \;&=\; [iQ_B, \bar{c}^a] \;=\; iB^a, \end{aligned} \tag{6}$$

where the BRST charge Q_B is given by

$$Q_B \;=\; \int d^3x\ \{B^a(D^{ab}_0 c^b) - \dot{B}^a c^a + i\frac{g}{2}f^{abc}\dot{\bar{c}}^a c^b c^c\} = Q^\dagger_B. \tag{7}$$

It can be shown

$$\delta F^a_{\mu\nu} \;=\; gf^{acd}F^c_{\mu\nu}c^d, \tag{8}$$

and writing $\delta F^a_{\mu\nu} = [iQ_B, F^a_{\mu\nu}]$, it follows that

$$[Q_B, F^a_{\mu\nu}] \;=\; igf^{adc}c^dF^c_{\mu\nu} \neq 0. \tag{9}$$

This property will be useful later when we discuss 'cluster property'.

The quantum equation of motion can be written as [7]

$$\partial^\mu F^a_{\mu\nu} \;=\; -gJ^a_\nu + \{Q_B, D^{ab}_\nu \bar{c}^b\}, \tag{10}$$

where

$$J^a_\mu \;=\; j^a_\mu + f^{abc}A^{\nu b}F^c_{\nu\mu} - \{Q_B, f^{abc}A^b_\mu \bar{c}^c\} + if^{abc}(\partial_\mu \bar{c}^b)c^c, \tag{11}$$

and

$$[Q_B, J^a_\mu] \;=\; -i\partial^\nu f^{abc}c^bF^c_{\mu\nu}. \tag{12}$$

3 Physical subspace

The total state vector space $\mathcal{V}$ in a covariant formulation of gauge theory necessarily contains negative norm states, that is, $\mathcal{V}$ has an indefinite metric. As the positivity of the metric is vital to the probabilistic interpretation, we need to define suitable subspace of $\mathcal{V}$ in such a way that the physical S-matrix, defined by restricting the total S-matrix to the physical subspace is unitary.

In non-Abelian gauge theory, the physical subspace $\mathcal{V}_{Phys} \subset \mathcal{V}$ is specified by the condition [9]

$$Q_B|Phys\rangle \;=\; 0;\; \mathcal{V}_{Phys} = \{|\phi\rangle; Q_B|\phi\rangle = 0\}. \tag{13}$$

As Q_B generates BRST transformations which are infinitesimal local gauge transformations, the above condition essentially expresses *the gauge invariance of the physical states belonging to* $\mathcal{V}_{Phys}$. The vacuum is annihilated by Q_B and so $|0\rangle \in \mathcal{V}_{Phys}$.

4 Observables in quantum YM theory

The physical space contains states with zero norm. We define $\mathcal{V}_{Phys}/\mathcal{V}_0$ and any zero norm state $|\chi\rangle$ in $\mathcal{V}_0$ is orthogonal to states in $\mathcal{V}_{Phys}$. The transition probability between physical states $T(\phi_1, \phi_2) = |(\phi_1, \phi_2)|^2$ has the property $T(\phi_1, \phi_2) = T(\phi_1 + \chi_1, \phi_2 + \chi_2)$. Besides transition probability, physical quantities to be measured must be such that any state $\in \mathcal{V}_0$ should make no physical effect in the measurement. To ensure this, if a zero norm state $|\chi\rangle \in \mathcal{V}_0$ were transformed by a physical quantity $\mathcal{R}$ in to $|\chi;\rangle = \mathcal{R}|\chi\rangle$, such that $(\phi, \chi') = (\phi, \mathcal{R}\chi) \neq 0$ for some $|\phi\rangle \in \mathcal{V}_{Phys}$, then the measurement of $\mathcal{R}$ could not be described consistently. So, it is required that the physical quantity to satisfy $(\phi, \mathcal{R}\chi) = 0$ - providing a definition for observable. As an example: consider P_μ. As Q_B is translation invariant, $[Q_B, P_\mu] = 0$. Let $P_\mu|\phi\rangle = |\psi\rangle$. Here $|\phi\rangle \in \mathcal{V}_{Phys}$; $Q_B|\phi\rangle = 0$. So, $Q_B|\psi\rangle = Q_B P_\mu|\phi\rangle = P_\mu(Q_B|\phi\rangle) = 0$. So $|\psi\rangle \in \mathcal{V}_{Phys}$. Since $|\chi\rangle \in \mathcal{V}_0$ is orthogonal to $|\phi\rangle$ and since $|\psi\rangle \in \mathcal{V}_{Phys}$, it follows that $(\chi, \psi) = 0$. So P_μ is an observable. We say that *if an operator $\hat{O}$ commutes with Q_B, then $\hat{O}$ is an observable.*

5 Confining potential and cluster property

In describing the potential mediated by fields, we need a potential not decreasing at infinity to confine quarks. This implies a failure of cluster property for the vacuum expectation value of two point function. We will elaborate this. The cluster property is

$$\langle 0|\phi_1(x_1)\phi_2(x_2)|0\rangle_{|x_1-x_2|\to\infty} \;\to \langle 0|\phi_1(x_1)|0\rangle\ \langle 0|\phi_2(x_2)|0\rangle. \tag{14}$$

Araki, Hepp and Ruelle [10] proved that the cluster property should hold in a Lorentz covariant local field theory with unique vacuum. In this case, the potential cannot be linearly rising. This produces a conflict in QCD as we expect a linear confining potential [11]. However, in non-Abelian gauge theory, a possible failure of cluster property has been pointed out by Strocchi [12]. In order to understand this, we recall the inequality derived by Araki, Hepp and Ruelle [10], on the assumption of covariance under translation, local commutativity, uniqueness of vacuum and spectral condition, that

$$|\langle 0|\phi_1(x_1)\phi_2(x_2)|0\rangle - \langle 0|\phi_1(x_1)|0\rangle\ \langle 0|\phi_2(x_2)|0\rangle| \;\leq\; c'[\xi]^{-2}[\xi]^{2N}(1 + \frac{|\xi^0|}{[\xi]}), \tag{15}$$

when there is no mass gap. Here $\xi = x_1 - x_2$ and N is a non-negative integer depending upon ϕ's. We first consider Abelian gauge theory (QED) where there is no mass gap. The role of Q_B is played by $B(x) = -\partial^\mu A_\mu(x)$. Using $[A_\mu(x), A_\nu(y)] = -i\eta_{\mu\nu}D(x-y)$, we have $[\partial_\mu A_\nu(x), \partial^\lambda A_\lambda(y)] = -i\partial_\mu\partial^\lambda\eta_{\nu\lambda}D(x-y)$ and $[\partial_\nu A_\mu(x), \partial^\lambda A_\lambda(y)] = -i\partial_\nu\partial^\lambda\eta_{\mu\lambda}D(x-y)$. It then follows that

$$[F_{\mu\nu}(x), B(y)] \;=\; 0, \tag{16}$$

and so $F_{\mu\nu}(x)$ *in QED is an observable.* Next, using

$$\langle 0|A_\mu(x)A_\nu(y)|0\rangle \;=\; \eta_{\mu\nu}F(x-y)+\partial_\mu\partial_\nu G(x-y), \tag{17}$$

we have

$$\langle 0|F_{\mu\nu}(x)F_{\sigma\rho}(y)|0\rangle = -\{\eta_{\mu\rho}\partial_\nu\partial_\sigma-\eta_{\nu\rho}\partial_\mu\partial_\sigma-\eta_{\mu\sigma}\partial_\nu\partial_\rho+\eta_{\nu\sigma}\partial_\mu\partial_\rho\}F(x-y). \tag{18}$$

Introduce now

$$F(f) \;=\; \int d^4x F_{\mu\nu}(x)\; f^{\mu\nu}(x), \tag{19}$$

where $f^{\mu\nu}(x)$ is some function in R^4. Then the state $F(f)|0\rangle$ will be in $\mathcal{V}_{Phys}$ since $F_{\mu\nu}(x)$ is an observable. Therefore

$$\langle 0|F^\dagger(f)F(f)|0\rangle \;\geq\; 0. \tag{20}$$

This implies that the Fourier transform of $\langle 0|F_{\mu\nu}(x)F_{\rho\sigma}(y)|0\rangle$ is a measure. So $N=0$. Thus, $F(x-y)\to[x-y]^{-2}$, which $\to 0$ as $|x-y|\to\infty$. Therefore the cluster property holds good and the potential also $\to 0$ as $|x-y|\to\infty$. Thus, the decrease of the potential is associated with the cluster property and in local field theory satisfying Wightman [13] axioms, the slowest decrease at infinity is like $\frac{1}{r}$, the Coulomb potential.

In the case of non-Abelian theory, like QCD, there is no mass gap. In here we have

$$[iQ_B, F^a_{\mu\nu}] \;=\; if^{abc}c^bF^c_{\mu\nu}, \tag{21}$$

and so $F^a_{\mu\nu}$ *is not an observable; gluon fields are not observables.* While the local charge in QED is an observable, in QCD the local BRST charges are not observable as $[Q^a.Q^b]=if^{abc}Q^c$. As a consequence of (9) or (21), the state

$$F(f)|0\rangle \;=\; \int d^4x F^a_{\mu\nu}(x) f^{a\;\mu\nu}(x)|0\rangle, \tag{22}$$

will not be in $\mathcal{V}_{Phys}$. This implies that the Fourier transform of $\langle 0|F^a_{\mu\nu}(x)F^b_{\rho\sigma}(y)|0\rangle$ will not be a measure and so $N\neq 0$. The cluster property fails. Thus, *the possibility of linear confining potential (not vanishing at spatial infinity) and quantum field theory are compatible.* The failure of the cluster property is thus related to the result that the quark and gluon states do not have asymptotic limit.

6 Gauge invariant states

A quark field with color index α satisfies (second relation in (6))

$$\delta\psi^\alpha = [Q_B, \psi^\alpha] \;=\; gc^a (t^\alpha)^\alpha_\beta \psi^\beta. \tag{23}$$

Since $[Q_B, \psi^\alpha] \neq 0$, ψ^α is not an observable. Let

$$|\alpha\rangle \;=\; \psi^\alpha |0\rangle, \tag{24}$$

from which it follows that $Q_B|\alpha\rangle \neq 0$ and so $|\alpha\rangle \notin \mathcal{V}_{Phys}$. As the S-matrix is defined for physical states, $|\alpha\rangle$ cannot be an asymptotic state. *This implies confinement of quarks.* Let us now consider a color singlet combination

$$\epsilon^{\alpha\beta\gamma}\psi^\alpha\psi^\beta\psi^\gamma \quad , \tag{25}$$

We will see this state to be asymptotic state. We then have

$$\begin{aligned} \delta_{BRST}(\epsilon^{\alpha\beta\gamma}\psi^\alpha\psi^\beta\psi^\gamma) &= \epsilon^{\alpha\beta\gamma}\left((\delta\psi^\alpha)\psi^\beta\psi^\gamma + \psi^\alpha(\delta\psi^\beta)\psi^\gamma + \psi^\alpha\psi^\beta(\delta\psi^\gamma)\right), \\ &= gc^a\epsilon^{\alpha\beta\gamma}\left((t^a)^\alpha_\delta\psi^\delta\psi^\beta\psi^\gamma + \psi^\alpha(t^a)^\beta_\delta\psi^\delta\psi^\gamma + \psi^\alpha\psi^\beta(t^a)^\gamma_\delta\psi^\delta\right). \end{aligned} \tag{26}$$

By explicitly computing this for $a = 1$ *to* 8 with Gell-Mann $SU(3)$ matrices [14] for t^a, we find that $\delta_{BRST}(\epsilon^{\alpha\beta\gamma}\psi^\alpha\psi^\beta\psi^\gamma) = 0$. which implies

$$[Q_B, \epsilon^{\alpha\beta\gamma}\psi^\alpha\psi^\beta\psi^\gamma] \;=\; 0. \tag{27}$$

Consequently, the operator $\epsilon^{\alpha\beta\gamma}\psi^\alpha\psi^\beta\psi^\gamma$ is an observable which represents baryon. Also, the state $\epsilon^{\alpha\beta\gamma}\psi^\alpha\psi^\beta\psi^\gamma|0\rangle$ has the property of being annihilated by Q_B and so $\in\; \mathcal{V}_{Phys}$. So, this is an asymptotic state and observable. This state represents the baryon.

7 Summary

We have considered the notion of confining potential and observables in the general framework of non-Abelian gauge field theory (QCD). Quarks and gluons as such are not observable as asymptotic states. The color singlet representing baryon (similar reasoning holds good for mesons) are observable as asymptotic states. Such color singlet states are gauge invariant. This is consistent with confinement property of QCD.

Acknowledgements

The author thanks A.P. Balachandran for his comments and reading of the manuscript.

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